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Master Thesis/Masterarbeit

Transmit beamforming in OFDM systems and cognitive radio networks

by/von
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17/05/10

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Declaration

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Abstract

Transmit beamforming is a technique used in MIMO systems to increase diversity in the communication. We consider a broadcast scenario (downlink) with several antennas at the transmitter and one antenna at each receiver. In this thesis we propose techniques that determine power allocation and beamformers in an OFDM scenario.

In Chapter 2 we make the scenario description and present some previous works in the single channel case. We describe two optimal transmit beamforming algorithms, and present one solution for the cognitive radio case.

In Chapter 3 we introduce the problematic of the OFDM optimization and justify the motivation behind it. We explain the interest in developing algorithms that allocate resources in a femtocell and consider the cognitive radio case. Within this scope we formulate two different approaches to our problem: one considers power minimization while having specific constraints on every subcarrier of each user, and the second minimizes power while considering only one constraint per user.

In Chapter 4 we develop two algorithms that suboptimally solve the problem with specific constraints on every subcarrier. For feedback signaling reasons, the users are imposed to use a single beamformer for all their subcarriers. Again, one of these solutions is extended to the cognitive radio case.

In Chapter 5 we propose a new algorithm that converges towards a feasible solution in the case we use Exponential Effective SINR Mapping (EESM) constraints on every user. We also discuss its optimality and extend it again to the cognitive case.

Finally, in Chapter 6 we present some simulations and results for these algorithms.

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Nomenclature

AWGN Additive White Gaussian Noise

BER Bit error rate

BLEP Block error probability

EESM Exponential Effective SINR Mapping

FDD Frequency Division Duplex

i.i.d. independent and identically distributed

LTE Long Term Evolution

MCS Modulation and Coding Scheme

MI-ESM Mutual Information based SINR Mapping

MIMO Multiple Input Multiple Output

MISO Multiple Input Single Output

OFDM Orthogonal Frequency-Division Multiplex

QoS Quality of Service

SINR Signal to Interference plus Noise Ratio

SNR Signal to Noise Ratio

TDD Time Division Duplex

ZMCS Zero Mean Circulary Symmetric

Chapter 1

Introduction

Wireless communications have evolved in the past years to serve many of the human necessities to transmit information in all kind of applications. From the long range distances such as satellite broadcasting services, to mobile networks or shorter range distances, they serve for many purposes and appear in all kind of devices. A great development in the use of these technologies has occurred, and more is expected due to the appearance of new applications and more demanding requirements. Higher transmission rates, wireless networks of sensors, or RFIDs ("Radio Frequency Identification") are some of the very active research areas that will have greater presence in the near future.

To solve many of these challenges, new techniques and methods have been proposed and studied. Transmission and reception with several antennas, generally known as MIMO communications, are now being widely used in many different scenarios and applications. These techniques allow to satisfy the QoS by creating diversity and increasing the transmission rate of the system (compared to a unique antenna for transmission and reception). New standards such as IEEE 802.11n, or the Long Term Evolution of UMTS (LTE) are some examples of commercial products which will appear in the coming years.

With the search of higher rates and to extend mobile coverage, the use of femtocells to cover indoor scenarios will probably grow in importance. These base stations provide wireless access in residential areas or small businesses in the licensed spectrum, and is of great interest in the 3G network and LTE. This interest lies in providing a better service to end-users, while reducing the capital expenditure and operating costs that would otherwise be required.

A new technology to transmit over the licensed spectrum without creating unnecessary interference appeared in the last years, known as *cognitive radio*. This approach is specially appropriate in femtocell scenarios, where it would help to reduce the frequency planning costs of mobile networks. It involves the usage of transmit beamforming in a MIMO transmission scheme, by knowing the channel, and adjusting the transmission beams to the users of interest.

The development of this thesis is based on this last scenario, where we study the implementation of algorithms that allocate resources (power and beamformers) on a broadcasting channel using OFDM modulation. In Chapter 2 we present the scenario description and introduce previous works in the topic. In Chapter 3 we explain the motivation and formulate the problems to solve. In Chapters 4 and 5 we present the algorithms we developed to solve these problems, and in Chapter 6 we present some simulations and results of their performance.

Chapter 2

Transmit beamforming overview

In this chapter we would like to present some background in the topic we are going to develop throughout the thesis, and introduce some previous works in which we based our work. This Chapter will mainly consider systems with only one channel, and we will extend these results in the subsequent Chapters to several transmission channels.

In the first section of this chapter we will introduce the scenario description, and in the following ones we present the algorithms that solve the beamforming problem in the single channel case.

2.1 Scenario description

In wireless communications, Multiple-Input-Multiple-Output (MIMO) is the use of multiple antennas at both the transmitter and receiver to improve communication performance. This technology offers significant increases in data throughput and link range without additional bandwidth or transmit power. It achieves this by having higher spectral efficiency (more bits per second per hertz of bandwidth) and link reliability or diversity (reduced fading). During this whole thesis, we will consider a multiuser Multiple-Input-Single-Output (MISO) system, where the base station (or transmitter) will have several antennas, and the user equipments (or receivers) will only have one. Please, refer to the bibliography in [1] for an introduction on MIMO communications.

Beamforming is one of the possible strategies to use when we have a MIMO system and we want to prioritize diversity over transmission rate. This strategy tries to create a user specific beam that will help the receiver to acquire sufficient Quality of Service (QoS) by cancelling the interference produced to other users while maintaining the signal of interest. These beams are formed by plugging some power distribution in the antenna array, and transmitting the same symbol in all antennas. This technique together with some signal processing at the receiver will help satisfy a certain QoS level and minimize the interference. It is important to note that all users transmit in the same frequency or channel, creating interference to each other. The purpose of this thesis is to design the algorithms that determine the beamformers and power allocation at the base station for the correct transmission of information.

A different approach in a MIMO system to create diversity would be to use space-time codes. The idea behind this strategy is to introduce some redundancy in the transmitted signal, both over time and space, and is mainly used when the transmitter has no knowledge of the propagation conditions. We will not cover this approach in the thesis, and would only like to mention it to the reader, for him to consider its use in other kind of scenarios. Again, you may refer to [1] for a further expansion on this topic.

Beamforming strategy varies if we consider transmission or reception in the network, and so, different problems arise in these two cases. The design of the beamformer at the receiver side will have to consider the signal quality for that specific user, whereas the design at the transmitter will have to deal with the overall system performance. We will deal with both approaches during the scope of the thesis, but will try to solve mainly the transmit beamforming strategy, which will call from now on “downlink problem”. The strategy from the receiver side, or “uplink problem”, will not be solved and presented as a separate problem, but will be included as part of the solution to the downlink one. Nevertheless, it will be easy to draw specific conclusions for the uplink formulation, sometimes because it will be solved as a middle step by means of duality, and others because the algorithm can easily be extended to cover both problems.

In this Chapter we will consider narrow-band systems, although some of these conclusions can be extended to wide-band systems [3]. It is not the purpose of this chapter to present those solutions, since the coming chapters will deal with wide-band channels by means of Orthogonal Frequency-Division Multiplex (OFDM).

As an additional requirement, the algorithms here presented will require complete knowledge of the channels at the transmitter, although approaches exist where the channel is partially known [6]. The channel characterizations can be estimated in several ways, depending on the strategy used for the downlink and uplink channels. For example, if both uplink and downlink use a Time Division Duplex (TDD), sharing the same frequency but different time slots, the base station could estimate the channels from the information transmitted by the receivers and extrapolate that information to the downlink. On the other hand, if a Frequency Division Duplex (FDD) scheme is used and the duplex distance is larger than the coherence bandwidth, the uplink and downlink channels will fade independently and the former approach would not work. In those cases a different solution can be used, simple but resource consuming mobile feedback, or hardware consuming alternatives such as matched arrays or spatial spectra estimation. For a deeper explanation of these alternatives, please refer to [3].

From now on, we will assume we only have one base station and the channels are perfectly known at the transmitter and receivers. We will also consider that the noise is white with power $\sigma^2 = \mathrm{E}\left[|n_i(t)|^2\right]$, the transmitted signals are narrow-band and the time dispersion of the radio link is small. The signal vector transmitted at the base station for user k will be $\mathbf{x}_k(t) = \mathbf{w}_k s_k(t)$, and the baseband received signal if we have a system with K different users and one base station will be:

$$r_i(t) = \sum_{k=1}^{K} \mathbf{h}_i^H \mathbf{x}_k(t) + n_i(t) = \sum_{k=1}^{K} \mathbf{h}_i^H \mathbf{w}_k s_k(t) + n_i(t) \qquad (2.1)$$

Here we present some of the variables and notation that will be used through the thesis:

$\mathbf{h}_i$ — Channel vector from the base station to the user i, including the channel gain.

$\mathbf{h}_{i,s}$ — Channel vector from the base station to the user i, in subcarrier s.

$\mathbf{R}_i$ — Channel correlation matrix from the base station to the user i, including the channel gain.

$\mathbf{R}_{i,s}$ — Channel correlation matrix from the base station to the user i, in subcarrier s.

$r_i(t)$ — Signal received at user i.

σ_i^2 — Noise level at user i.

$\mathbf{w}_i$ — Beamforming vector for transmission from the base station to user i.

$\mathbf{w}_{i,s}$ — Beamforming vector for transmission from the base station to user i, in subcarrier s.

$s_i(t)$ — Signal intended for user i.

$\mathbf{x}_k(t)$ — Signal vector transmitted at the base station.

K — Total number of users.

N — Total number of subcarriers.

It is difficult to obtain good estimates for the instantaneous downlink channel vectors $\mathbf{h}_i$, but it is reasonable to have access to the channel correlation matrix, estimated as an average over the fast fading $\mathbf{R}_i = \mathrm{E}\left[\mathbf{h}_i \mathbf{h}_i^H\right]$. Therefore, in our algorithms we will deal with this parameter, rather than the channel vectors.

2.2 Optimal transmit beamforming

There are many ways on how to define *optimal beamforming*, but we will deal with the one that tries to minimize power while satisfying some user QoS requirements. We will express the QoS requirement as a lower threshold γ_i on the received Signal to Interference plus Noise Ratio (SINR) at each user. Our problem can then be formulated as

$$\begin{array}{cl} \min & \text{Total transmitted power} \\ s.t. & \frac{\text{Received signal power at user } i}{\text{Received interference+noise}} \geq \gamma_i \end{array}$$

Other techniques consider the maximization of the worst SINR, while satisfying some power constraint. This approach can be checked in [13, 7], but we will not deal with it along the thesis. In addition to that, we would like to mention that the information presented in this section has been gathered from [3].

With our previous formulation, in order to represent it with mathematical terms, we can express (2.1) in this way:

$$r_i(t) = \underbrace{\mathbf{h}_i^H \mathbf{w}_i s_i(t)}_{desired\,signal} + \underbrace{\sum_{n \neq i} \mathbf{h}_i^H \mathbf{w}_n s_n(t)}_{interference} + \underbrace{n_i(t)}_{noise} \tag{2.2}$$

It is straightforward, that with this formulation the SINR at the receiver is then

$$\text{SINR}_i = \frac{\mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i}{\sum_{n \neq i} \mathbf{w}_n^H \mathbf{R}_i \mathbf{w}_n + \sigma_i^2} \tag{2.3}$$

where we used the channel correlation matrix, instead of the instantaneous channel vector.

The transmitted power at the base station is $E\left[\|\mathbf{w}_i s_i(t)\|^2\right] = \mathbf{w}_i^H \mathbf{w}_i$, and the overall beamforming can be presented as

$$\begin{aligned} &\min \sum_{i=1}^{K} \mathbf{w}_i^H \mathbf{w}_i \\ &s.t. \frac{\mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i}{\sum_{n \neq i} \mathbf{w}_n^H \mathbf{R}_i \mathbf{w}_n + \sigma_i^2} \geq \gamma_i \end{aligned} \tag{2.4}$$

This is a quadratic optimization problem with non-convex constraints, which as it stays is NP-complete and not solvable in reasonable time. However, it can be transformed and expressed in an equivalent way that will allow us to solve it efficiently. We present two algorithms in the coming subsections that will solve it.

Note from this formulation that for the optimal $\mathbf{w}_i$ all constraints will hold with equality. To understand this, suppose the optimal solution fulfills one of the constraints with strict inequality. Then we could diminish the power of that beamformer until the constraint is satisfied with equality, and that would reduce the interference produced to other users. However the new found solution uses less power than the starting one, and therefore states a contradiction. From this we can establish that all constraints are satisfied with equality at the optimum. Additionally to that statement, we should notice that since all the beamformers are involved in all the constraints, the solution must be calculated centrally.

This formulation (2.4) is used for the downlink problem, when we want to determine the beamformer at the base station to transmit to the different mobiles/users. Now, we will consider the uplink problem, where all the users transmit simultaneously and the base station wants to decode the different user signals. Since the users only have one antenna, the received vector signal at the base station in the uplink is

$$\mathbf{r}_i(t) = \sum_{i=1}^{K} \mathbf{h}_i^{UL} s_i(t) + \mathbf{n}_i(t) \tag{2.5}$$

When we multiply by the beamformer to decode the signal, we get

$$y_i(t) = \mathbf{r}_i^H(t) \cdot \mathbf{w}_i = \underbrace{\left(\mathbf{h}_i^{UL}\right)^H \cdot \mathbf{w}_i s_i(t)}_{desired\,signal} + \underbrace{\sum_{n \neq i} \left(\mathbf{h}_n^{UL}\right)^H \cdot \mathbf{w}_i s_n(t)}_{interference} + \underbrace{\mathbf{n}_i(t) \cdot \mathbf{w}_i}_{noise} \tag{2.6}$$

Finally, the SINR obtained for that user will be

$$\mathrm{SINR}_i = \frac{\mathbf{w}_i^H \mathbf{R}_i^{UL} \mathbf{w}_i}{\mathbf{w}_i^H \left(\sum_{n \neq i} \mathbf{R}_n^{UL} + \sigma_i^2 \mathbf{I}\right) \mathbf{w}_i} \tag{2.7}$$

As before, we will consider the SINR as a measure of the QoS provided to the specific user. Note in these last expressions, that the channel vectors and correlation matrices do not necessarily have to be the same as in the downlink case. In the future sections we will omit the specific reference "UL" (uplink) for notation simplicity.

2.2.1 An algorithm based on power control

The idea followed in this algorithm is to reformulate the downlink problem as an equivalent uplink problem, and then solve the last one iteratively. This result is based on the following lemma:

Lemma 1. *If* $\{\mathbf{w}_i\}$ *are the optimal beamformers for the downlink beamform-*

ing problem (2.4) *and* $\{\mathbf{u}_i\}$ *is the optimal solution for the following problem*

$$\begin{aligned} &\min \sum_{i=1}^{K} \rho_i \\ &s.t. \frac{\mathbf{u}_i^H \rho_i \mathbf{R}_i \mathbf{u}_i}{\mathbf{u}_i^H \left(\sum_{n \neq i} \rho_n \gamma_n \mathbf{R}_n + \mathbf{I} \right) \mathbf{u}_i} \geq 1 \\ &\|\mathbf{u}_i\|^2 = 1 \end{aligned} \tag{2.8}$$

then for all the beamformers, $\mathbf{w}_i = \sqrt{p_i}\mathbf{u}_i$ *for some positive constants* p_i.

We will not show proof of this result now, although it can be checked in reference [3]. Later in Section 4.2 we make use of a similar result that extends the lemma, and we will prove both.

The reformulated problem in (2.8) has the same structure as an uplink SINR (2.7), where we have unity norm $\{\mathbf{u}_i\}$ beamformers, and power minimization $\{\rho_i\}$ at the mobiles. Note, that this reformulated problem is different to the *true uplink problem*, and therefore we will call it *equivalent uplink problem*.

The algorithm is based on an iterative process where the beamformer that maximizes the SINR is determined, and then the power that minimizes the total power for that beamformer is updated. These steps are repeated until the variables do not change above a given error. Convergence is guaranteed if a feasible solution exists, and proof of it can be checked in [11, 16]. We present in table 2.1 the steps of the algorithm.

Finally, note that the maximum beamformer $\mathbf{u}_i$ that maximizes μ_i is given by the maximum eigenvector of the generalized eigenvalue problem

$$\rho_i(t)\,\mathbf{R}_i\mathbf{u}_i = \mu_i \left(\sum_{k \neq i} \rho_n(t)\,\gamma_n \mathbf{R}_n + \mathbf{I} \right) \mathbf{u}_i \tag{2.12}$$

We will make use of this result in Section 4.2 to present another version of the algorithm to cover a different formulation of the problem.

2.2.2 An algorithm based on semidefinite optimization

The next approach, derived in [2, 3], will consider time-varying beamformers, taken from some distribution that will have correlation matrices $\mathbf{W}_i = \mathrm{E}\left[\mathbf{w}_i(t)\,\mathbf{w}_i^H(t)\right]$. This will add more degrees of freedom in the search of the beamformers, and because of the circularity property of the trace $\mathrm{Tr}\{\mathbf{AB}\} = \mathrm{Tr}\{\mathbf{BA}\}$ we may transform the initial quadratic constraints from problem (2.4) to linear ones. It follows $\mathbf{w}^H\mathbf{R}\mathbf{w} = \mathrm{Tr}\left\{\mathbf{R}\mathbf{w}\mathbf{w}^H\right\} = \mathrm{Tr}\{\mathbf{RW}\}$, and by expressing all constraints in linear terms, we get

Algorithm 2.1 Based on power control

1. Initialize $\rho_i = 1$ for $i = 1, 2, \dots K$.

2. For $t = 1, 2, \dots$ until convergence, iterate the following steps

   Beamformer update: find

$$\mu_i = \max_{\|\mathbf{u}_i\|=1} \frac{\rho_i(t)\,\mathbf{u}_i^H \mathbf{R}_i \mathbf{u}_i}{\mathbf{u}_i^H \left(\sum_{n\neq i} \rho_n(t)\,\gamma_n \mathbf{R}_n + \mathbf{I}\right)\mathbf{u}_i} \tag{2.9}$$

   and the corresponding vector $\mathbf{u}_i$ for $i = 1, 2, \dots K$.

   Power control update:

$$\rho_i(t+1) = \frac{1}{\mu_i}\rho_i(t) \tag{2.10}$$

3. After convergence,

$$\begin{aligned}
\boldsymbol{\eta} &= \left[\gamma_1\sigma_1^2, \dots, \gamma_K\sigma_K^2\right] \\
[\mathbf{F}]_{i,n} &= \begin{cases} \mathbf{u}_i^H \mathbf{R}_i \mathbf{u}_i & i = n \\ -\gamma_i \mathbf{u}_n^H \mathbf{R}_i \mathbf{u}_n & i \neq n \end{cases} \\
\mathbf{p} &= \mathbf{F}^{-1}\boldsymbol{\eta} \\
\mathbf{w}_i &= \sqrt{p_i}\,\mathbf{u}_i
\end{aligned} \tag{2.11}$$

$$\begin{aligned}
&\min \sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{W}_i\} \\
&s.t.\ \mathrm{Tr}\{\mathbf{R}_i\mathbf{W}_i\} - \gamma_i \sum_{n\neq i} \mathrm{Tr}\{\mathbf{R}_i\mathbf{W}_n\} \geq \gamma_i \\
&\quad \mathbf{W}_i = \mathbf{W}_i^H \\
&\quad \mathbf{W}_i \succeq 0
\end{aligned} \tag{2.13}$$

We made explicit statement that matrices $\mathbf{W}$ remain hermitian and positive semidefinite, since they are considered correlation matrices. The notation $\mathbf{W}_i \succeq 0$ means that $\mathbf{W}$ is positive semidefinite. The problem (2.13) as it stays is convex, and then solvable with convex optimization tools. For this thesis we used `CVX` to solve them [10].

When introducing time-varying beamfomers and their correlation matrix $\mathbf{W}$, we changed our original problem to a different one. The problem could have stayed equivalent, if we forced $\mathbf{W}$ to have rank one. However, the rank $\{\mathbf{W}\} = 1$ constraint is nonconvex, but by relaxing the constraint we get a convex problem. The surprising thing in this case is that this omission is no relaxation at all, because the convex problem we obtain is in fact equivalent to (2.4), and the solution we get from (2.13) will probably have rank one. For proof of this statement refer to [3].

Finally, to obtain the beamformers we need to span the vectors that form the correlation matrices $\mathbf{W}_i$, where we choose the one with highest associated eigenvalue. To find the powers, we use the same procedure as in (2.11).

It is important to note that this algorithm allows the inclusion of other constraints, such as power limits as long the constraint can be expressed within the formulation of convex problems. In these cases, there is no further guarantee that $\mathbf{W}$ will hold rank one and such a solution could be interpreted as a non-obtainable solution with fix-time beamformers. In coming sections and Chapters we will also use this relaxation, and we will note that there is no guarantee that rank $\{\mathbf{W}\} = 1$ will hold.

2.3 Cognitive Radio

Cognitive radio is a new paradigm for wireless communication in which either a network or a wireless node changes its transmission or reception parameters to communicate efficiently avoiding interference with licensed or unlicensed users. It supports exploitation of the spectrum by the unlicensed users (also known as secondary users) to communicate in the licensed bands when they are unoccupied by the licensed holders (or primary users). It is the objective of this thesis to provide solutions to the problem of cognitive radio in the different formulations we will encounter in the coming Chapters. For this reason, we include this section as introduction towards what we will develop in coming algorithms. See [6, 7] for some references on the topic of cognitive radio.

Transmit beamformers can be designed by setting constraints on the interference level of the licensed users and SINRs on the secondary users. We will consider an approach where we have L primary users and K secondary users. The base station will be equipped with N antennas, while both the primary and secondary users will have only one. We will assume the same notation as in Section (2.1), and the same SINRs on secondary users as in equation (2.3).

Since the cognitive users (secondary users) share the same frequency band, they may cause interference to the primary users. The interference signal at the l th primary user can be written as,

$$r_l = \sum_{i=1}^{K} \mathbf{h}_l^H \mathbf{w}_i s_i(t) \tag{2.14}$$

where $\mathbf{h}_l$ represents the channel from the base station to the primary user. The maximum interference power allowed to the primary user, named ϵ_l for $l = 1, 2 \ldots L$, will be

$$P_l = \sum_{i=1}^{K} \mathbf{w}_i^H \mathbf{R}_l \mathbf{w}_i \leq \epsilon_l \tag{2.15}$$

where we used the channel covariance matrix $\mathbf{R}_l$ instead of the instantaneous channel vector.

The problem formulation with all the constraints will be,

$$\begin{aligned} &\min \sum_{i=1}^{K} \mathbf{w}_i^H \mathbf{w}_i && (2.16) \\ &s.t. \frac{\mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i}{\sum_{n \neq i} \mathbf{w}_n^H \mathbf{R}_i \mathbf{w}_n + \sigma_i^2} \geq \gamma_i, && i = 1, 2 \ldots K \\ &\sum_{i=1}^{K} \mathbf{w}_i^H \mathbf{h}_l \mathbf{h}_l^H \mathbf{w}_i \leq \epsilon_l, && l = 1, 2 \ldots L \end{aligned}$$

To solve the problem, a similar approach to the one in Section 2.2.2 can be followed, converting the original quadratic problem into a convex one,

$$\begin{aligned} &\min \sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{W}_i\} && (2.17) \\ &s.t. \mathrm{Tr}\{\mathbf{R}_i \mathbf{W}_i\} - \gamma_i \sum_{n \neq i} \mathrm{Tr}\{\mathbf{R}_i \mathbf{W}_n\} \geq \gamma_i, && i = 1, 2 \ldots K \\ &\sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{H}_l \mathbf{W}_i\} \leq \epsilon_l && l = 1, 2 \ldots L \\ &\mathbf{W}_i = \mathbf{W}_i^H \\ &\mathbf{W}_i \succeq 0 \end{aligned}$$

The obtained correlation matrix $\mathbf{W}_i$ is also guaranteed to have rank one, and the same conclusions as in Section 2.2.2 apply. The beamformer $\mathbf{w}_i$ will again be found as the eigenvector that expand the matrix $\mathbf{W}_i$.

Chapter 3

Problem statement

In this chapter we explain the motivation and purpose of the thesis, and cite the mathematical formulations we try to solve. In the first section, we establish the interest to investigate our problem in a femtocell scenario, and within the cognitive radio technology. In the next section, we discuss the validity of the mathematical formulations we use, and the alternatives we deal with.

3.1 Motivation and purpose

A femtocell is a small cellular base station, typically designed for use in a home or small business. It connects to the service provider's network via broadband (such as DSL or cable) and allows service providers to extend service coverage indoors. It enables capacity equivalent to a full 3G network sector at very low transmit powers, dramatically increasing battery life of existing phones, without needing to introduce WiFi enabled handsets. It operates in licensed spectrum, so the deployment of equipment must meet the strict requirements of the licenses. Therefore, it has implications in frequency and cellular planning, since an unexpectedly located access point base station could interfere with other closely-located base stations.

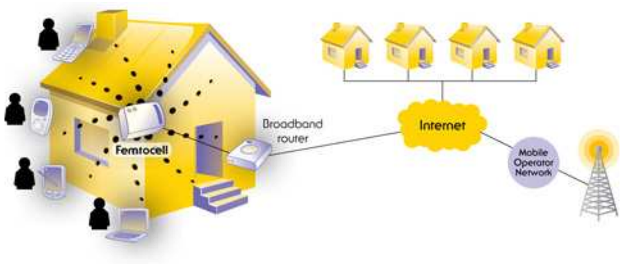


Figure 3.1: Diagram of a Femtocell scenario

In order to avoid interference caused to macrocell users or base stations, the cognitive radio approach could provide a liable solution to the problematic of frequency planning. As explained in Section 2.1, the base station transmits to the unlicensed users (or secondary users) without causing interference to the macrocell users (primary users). What the base station does is to conform the transmit beam to reach the user of interest, while minimizing the interference caused to the primary users, and satisfying some QoS to the rest of secondary users. In Figure 3.2 we see the downlink scenario that represents the interference caused to these users.

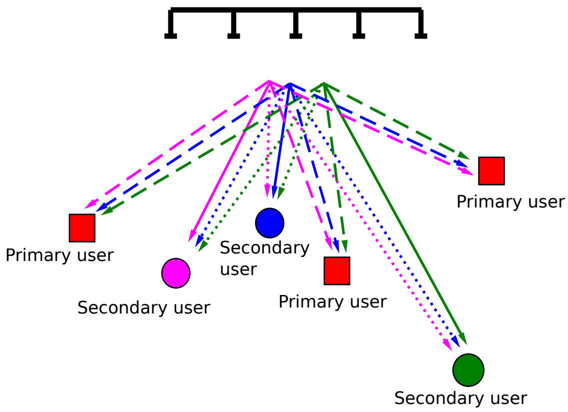


Figure 3.2: Interference caused to PU and SU

OFDM is a frequency-division multiplexing scheme utilized as a digital multi-carrier modulation method, where a large number of closely-spaced orthogonal sub-carriers are used to carry data. Each sub-carrier is modulated with a conventional modulation scheme (such as quadrature amplitude modulation or phase-shift keying) at a low symbol rate, maintaining total data rates similar to conventional single-carrier modulation schemes in the same bandwidth. Please refer to [1, 8] for a description of its mathematical characterization.

OFDM has developed into a popular scheme for wideband digital communications, being adopted in many standards such as in the Long Term Evolution of UMTS. Therefore, a growing interest in combining the MIMO techniques and OFDM modulation scheme has aroused in the past years (also dealing with the cognitive radio approach). In Figure 3.3 we see the transmission and reception scheme used in a MIMO-OFDM system with a beamforming strategy. Every symbol will first be scaled by the beamforming vector, and each output will enter the different OFDM modulators. Note, that in general the beamformer will be different at each subcarrier. At the receiver side, the user will only have one antenna, although the Figure shows the general case.

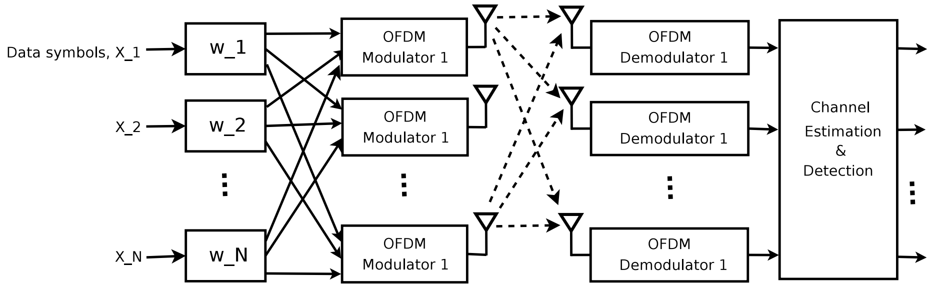


Figure 3.3: MIMO-OFDM transmission and receiver scheme

The purpose of this thesis is to investigate the use of algorithms that will determine the power allocation and beamformers in a MIMO-OFDM system, also with the problematic on how to allocate these resources on the spectrum. The previous works presented in Chapter 2 considered the single channel case, and we will propose an extension to those algorithms on the OFDM case, where we have multiple subcarriers. We will discuss their suboptimality, run simulations and present our conclusions. For this thesis, we will assume that the cyclic prefix of the OFDM system is larger than the channel dispersion, so that orthogonality between symbols is preserved.

As mentioned before in the MIMO-OFDM scheme, different beamformers could be used in every subcarrier to conform the beam to the channel behavior in different frequencies. However, for signaling purposes it is of great interest to use known beamformers at the receiver side, although they may not have optimal behavior. Several possibilities have been proposed, such as using beamformers from a codebook [14], using one beamformer at each cluster of subcarriers [5], or using a technique called smooth-beamforming that does not decorrelate the beamformed channels [12]. The interest in using one of these techniques is to assure that the subcarriers at the receiver will maintain correlation, allowing easy channel estimation on the nearby subcarriers. Otherwise, the subcarriers would decorrelate by using independent beamformers, increasing the necessity of more pilot signals to estimate the channels.

As an additional objective to extend the algorithms to the OFDM case, we decided to study the use of fixed beamformers on clusters of beamformers. In previous approaches they used one beamformer per cluster, where the subcarriers were correlated. In our study, we will deal with independent frequency clusters, and search for a good beamformer that works on several clusters, satisfying the given constraints.

3.2 Problem formulation in the OFDM case

When deciding how to formulate the problem in the OFDM case we considered several alternatives. The first one was to minimize power using the algorithms from sections 2.2.1 and 2.2.2. These algorithms would be applied independently to the different subcarriers, finding the optimal beamformers and power allocation to each of them. This approach has several drawbacks. The first one is that these beamformers would decorrelate the channels, increasing the number of pilot signals necessary to estimate the channels at the receiver. The second one is that we would need to set individual constraints on every user and every subcarrier to find the beamformers, and do the spectrum planning somehow.

To avoid the first drawback, we need to find beamformers that work in a wide band of the spectrum. By including the subcarriers in *clusters*, we try to find the beamformers that work on several of them. This justifies that we consider the clusters independent from each other, as they represent sets of subcarriers wide apart from each other in the transmit spectrum.

The second drawback about specifying individual SINR constraints to each user and subcarrier, opens the question on how to represent the desired QoS to the different users. We decided that capacity constraints, and effective SINR constraints could represent globally the QoS of the users. In the next section we present some initial approaches we had to reject, that help understand the problematic. In Section 3.2.3 we explain the concepts we finally use in the rest of the thesis.

3.2.1 Initial approaches

The first idea on how to represent a *user general constraint* was to make an averaged SINR over all subcarriers, making it greater than a given lower bound

$$\frac{1}{N}\sum_{s=1}^{N}\mathrm{SINR}_{i,s} \geq \gamma_i \quad i \in \{1, 2 \ldots K\} \tag{3.1}$$

where the index i represents the user i out of K total users, the index s the different subcarriers of the total N, and the SINR is defined as in Section 2.2. The reason for not being valid, is that satisfying that the average SINR is above some value does not describe the performance of an OFDM system and also, does not represent the intended QoS provided to the user. However, this idea is somehow exploited in the effective SINR mapping method, which we will later introduce.

A second idea we investigated, was to have a constraint on the total signal power of all subcarriers, divided by all interference and noise,

$$\frac{\sum_{s=1}^{N} \text{power on subcarrier}_s}{\sum_{s=1}^{N} \text{interferenc on subcarrier}_s + \text{noise}} \tag{3.2}$$

$$\frac{\sum_{s=1}^{N} \mathbf{w}_i^H \mathbf{R}_{i,s} \mathbf{w}_i}{\sum_{s=1}^{N} \left(\sum_{n \neq i} \mathbf{w}_n^H \mathbf{R}_{i,s} \mathbf{w}_n + \sigma_{i,s}^2 \right)} \geq \gamma_i \tag{3.3}$$

where $\mathbf{R}_{i,s}$ represents the channel covariance matrix of user i on subcarrier s, $\mathbf{w}_i$ represents the beamformer used by user i (note that there is only one beamformer per user), and $\sigma_{i,s}^2$ represents Additive White Gaussian Noise (AWGN). This approach can easily be solved by summing the subcarrier covariance matrices of the users, and using the algorithms of sections 2.2.1 and 2.2.2 to find the beamformers.

What this approach implies, is that we transmit a fixed amount of power on all subcarriers of user i, and we expect that the total amount of received power is above the total interference and noise scaled by γ_i. What we noticed, is that with this approach the problem quickly turns unfeasible, being bounded the number of subcarriers that can be handled by one beamformer. After this, we decided to separate the allocation of power from the beamformer strategy, so that the power is not distributed equally on all subcarriers. Here,

$$\frac{\sum_{s=1}^{N} p_{i,s} \mathbf{u}_i^H \mathbf{R}_{i,s} \mathbf{u}_i}{\sum_{s=1}^{N} \left(\sum_{n \neq i} p_{n,s} \mathbf{u}_n^H \mathbf{R}_{i,s} \mathbf{u}_n + \sigma_{i,s}^2 \right)} \geq \gamma_i \tag{3.4}$$

$$\mathbf{u}_i^H \mathbf{u}_i = 1$$

the variables $p_{i,s}$ indicate the power of user i to transmit in subcarrier s, and $\mathbf{u}_i$ is a unity norm vector.

However, this formulation is inappropriate, and we will show it with a counter-example. Suppose the scenario of two subcarriers (or two clusters, using the terminology from before) and two users. The optimal strategy in this case would be that both users transmit in different subcarriers, without causing any interference to each other, and without needing a specific beamformer that cancels any interference. Nevertheless, the formula from above will assume there is interference and it will consider the noise from both subcarriers, although the user does not transmit on the second channel. This is enough reason to deviate from the optimum, since the formula tries some interference minimization, where there is none. You may check this explanation on table 3.1.

3.2.2 Formulation with individual constraints

In this section we consider the mathematical formulation to find the resource allocation of power and beamformers, when the beamformers are kept fixed in all subcarriers. For this problem, we will assume that SINR constraints are given

POWERS	User 1	User 2
Subcarrier 1	$p_{1,1}$	0
Subcarrier 2	0	$p_{2,2}$

(a)

User 1	$\frac{p_{1,1}\mathbf{u}_1^H\mathbf{R}_{1,1}\mathbf{u}_1}{p_{2,2}\mathbf{u}_2^H\mathbf{R}_{1,2}\mathbf{u}_2+\sigma^2}$
User 2	$\frac{p_{2,2}\mathbf{u}_2^H\mathbf{R}_{2,2}\mathbf{u}_2}{p_{1,1}\mathbf{u}_1^H\mathbf{R}_{2,1}\mathbf{u}_1+\sigma^2}$

(b)

Table 3.1: Counter-example

on every user and on every subcarrier, and we will call this formulation *case with individual constraints.* In the next section we will specify the problem giving only one constraint per user, and we will call it *case with general constraints.*

Having said all this, and knowing our starting point is to minimize power, the problem to solve reads

$$\begin{aligned}
&\min \sum_{i=1}^{K}\sum_{s=1}^{N} p_{i,s} && (3.5)\\
&s.t.\ \frac{p_{i,s}\mathbf{u}_i^H\mathbf{R}_{i,s}\mathbf{u}_i}{\sum_{n\neq i} p_{n,s}\mathbf{u}_n^H\mathbf{R}_{i,s}\mathbf{u}_n+\sigma_{i,s}^2} \geq \gamma_{i,s} \quad i\in\{1,2\ldots K\}\\
&\mathbf{u}_i^H\mathbf{u}_i = 1 \quad s\in\{1,2\ldots N\}\\
&p_{i,s}\geq 0
\end{aligned}$$

where $p_{i,s}$ is the power of user i on subcarrier s, $\gamma_{i,s}$ represents the individual constraint on the subcarrier s, and $\mathbf{u}_i$ are unity norm vectors. Note there is only one beamformer per user, and $K\cdot S$ SINR constraints. Table 3.2 shows the aforementioned constraints.

	User 1	User 2	...	User K
Subcarrier 1	$\gamma_{1,1}$	$\gamma_{2,1}$		$\gamma_{K,1}$
Subcarrier 2	$\gamma_{1,2}$	$\ddots$		$\vdots$
...	$\vdots$			
Subcarrier N	$\gamma_{1,N}$			$\gamma_{K,N}$

Table 3.2: Individual constraints

In Chapter 4 we will propose two algorithms that solve this problem suboptimally. One of them will be based on semidefinite optimization, while the other will be an iterative algorithm based on eigenvalue decomposition approaches. To solve this second algorithm we will look into an equivalent uplink formulation.

3.2.3 Formulation with general constraints

In the single channel case, SINR is a good measure of QoS, since it can be related to a Bit Error Rate (BER), or to a capacity value. For this reason, a similar parameter that could represent the performance of the OFDM channel was required. The effective SINR mapping method [15] turned out to represent the characteristics of multiple sub-carrier SINR, and therefore we decided to represent the general constraints with this measure. A different approach was to use capacity constraints on each user, but this method presented some difficulties that made it more difficult to solve.

3.2.3.1 Effective SINR Mapping method

The effective SINR measure is a mapping method that helps represent the characteristics of multiple sub-carrier SINR [15]. This concept aroused because of the necessity to evaluate an OFDM system performance with only one parameter, due to the high load simulations would require, with all possible subcarrier states. The main objective of ESM is to find a value that is able to go from an instantaneous SINR at every subcarrier, to a corresponding block error probability (BLEP). This mapping will use either a look-up table, or an approximate analytical expression. This procedure is shown in Figure 3.4.

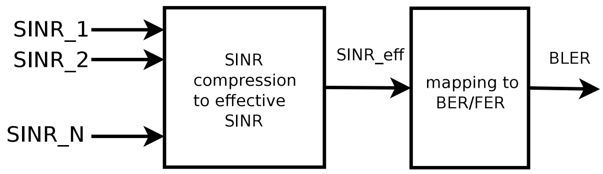


Figure 3.4: Principle of the effective SINR mapping

Currently, there are two approaches in the literature, namely Exponential Effective SINR Mapping (EESM) and Mutual Information based SINR Mapping (MI-ESM). The first one is appropriate when all subcarriers on the mobile terminal have to use the same modulation and coding scheme (MCS). The second method is more advanced, and is suitable for adaptive modulation and coding schemes in the different subcarriers. We will only explain EESM, as it is the one we are going to use in our problem, due to the complexity the other method involves. The exponential effective SINR is represented by equation 3.6, where β is a scaling factor used to adjust the compression function to adapt the mismatch between the predicted BLEP and the real one.

$$\mathrm{SINR}_{eff} = -\beta \ln \left[\frac{1}{N} \sum_{s=1}^{N} \exp\left(-\frac{1}{\beta} \mathrm{SINR}_s \right) \right] \tag{3.6}$$

Note again that EESM is used only for fixed modulation and coding scheme on all subcarriers, and β is a fixed parameter that will only depend on them. For a set of values of β depending on these parameters, you should check [15]. Finally, observe this expression involves the inverse function (logarithm) of an average of negative exponentials, whose arguments are the SINR_s where the index s indicates the different subcarriers.

3.2.3.2 Problem formulation with EESM constraints

The problem with EESM constraints then reads

$$\min \sum_{i=1}^{K} \sum_{s=1}^{N} \mathbf{w}_{i,s}^{H} \mathbf{w}_{i,s} \tag{3.7}$$
$$s.t. \; -\beta \ln \left[\frac{1}{N} \sum_{s=1}^{N} \exp \left(-\frac{1}{\beta} \frac{\mathbf{w}_{i,s}^{H} \mathbf{R}_{i,s} \mathbf{w}_{i,s}}{\sum_{n \neq i} \mathbf{w}_{n,s}^{H} \mathbf{R}_{i,s} \mathbf{w}_{n,s} + \sigma_{i,s}^{2}} \right) \right] \geq \gamma_i$$
$$i \in \{1, 2 \ldots K\}$$

where we have K constraints (total number of users). Note β is a fixed parameter that depends only on the MCS. Again, γ_i represents the effective SINR constraint each secondary user must achieve.

3.2.3.3 Problem formulation with capacity constraints

The channel capacity formula for each user is the sum of the subcarrier capacities, being a function of the SINR on the different subcarriers (indicated by index s).

$$C_i = \sum_{s=1}^{N} \frac{1}{2} \log \left(1 + \mathrm{SINR}_s \right) \tag{3.8}$$

The problem with capacity constraints is then

$$\min \sum_{i=1}^{K} \sum_{s=1}^{N} \mathbf{w}_{i,s}^{H} \mathbf{w}_{i,s} \tag{3.9}$$
$$s.t. \; \frac{1}{2} \sum_{s=1}^{N} \log \left(1 + \frac{\mathbf{w}_{i,s}^{H} \mathbf{R}_{i,s} \mathbf{w}_{i,s}}{\sum_{n \neq i} \mathbf{w}_{n,s}^{H} \mathbf{R}_{i,s} \mathbf{w}_{n,s} + \sigma_{i,s}^{2}} \right) \geq \gamma_i$$
$$i \in \{1, 2 \ldots K\}$$

where we considered one beamformer per subcarrier and user.

However, we did not find a solution to this problem and it remains unsolved. In Section 5.3 we explain briefly the approach we followed, and the reason why it did not converge.

Chapter 4

Algorithms with individual constraints

In this chapter we present two suboptimal algorithms that solve 3.5. The first one is based on semidefinite programming, and can be easily extended to solve the cognitive radio case as in Section 2.3. The second one is formulated as an equivalent uplink problem, and then solved iteratively until conversion. In Chapter 6 we present some simulations and results. The minimization problem is

$$\begin{aligned}
&\min \sum_{i=1}^{K} \sum_{s=1}^{N} p_{i,s} \\
&s.t.\ \frac{p_{i,s}\mathbf{u}_i^H \mathbf{R}_{i,s}\mathbf{u}_i}{\sum_{n\neq i} p_{n,s}\mathbf{u}_n^H \mathbf{R}_{i,s}\mathbf{u}_n + \sigma_{i,s}^2} \geq \gamma_{i,s} \quad i \in \{1, 2 \ldots K\} \\
&\quad \mathbf{u}_i^H \mathbf{u}_i = 1 \qquad\qquad\qquad\qquad s \in \{1, 2 \ldots N\} \\
&\quad p_{i,s} \geq 0
\end{aligned} \tag{4.1}$$

We observe the problem is nonconvex, with quadratic constraints in the numerator and denominator, and variables $p_{i,s}$ and $\mathbf{u}_{i,s}$ multiplying to each other. This fact makes it uneasy to search for a global solution of the problem, and prevents to use an approach like the one in Section 2.2. These variables relate as $\mathbf{w}_{i,s} = p_{i,s}\mathbf{u}_i$ where each user has a single beamformer and different power allocation on each subcarrier, where the index i represents the user of interest, and s the subcarrier in consideration.

4.1 Algorithm based on semidefinite optimization

In this section we present an algorithm that solves problem 4.1 suboptimally. The idea of this algorithm is to fix one of the variables to compute while optimizing the other one, and in the next step fix the other variable and optimize the first one. At each step, power minimization is made so that we always decrease the power being used. The idea is that in the first step power is distributed among the user, and in the second the power is distributed among the subcarriers. Both steps are iterated until convergence.

In addition to that, the result at each step has to be normalized by the user power, so that in the next step we keep minimizing the overall power. We observed that the result depends on the initialization point, making the first power allocation of great importance. Table 4.1 shows the steps of the algorithm to follow. Notice that we introduce the correlation matrices of the beamformers as we did in Section 2.2.2, and relax the rank $\{\mathbf{W}_i\} = 1$ constraint. The problem is still nonconvex because of the multiplication of two variables. However, if we fix one of the variables, the obtained equation will be linear and therefore solvable. In fact the relaxed problem will be equivalent to the original problem, and it is the same proof as in appendix A of [3].

In step 1 we search for an initialization point, taking into account the channel gain, the noise and the $\gamma_{i,s}$ requirements. This initialization vector is the one that solves the problem when considering only the signal to noise ratio (SNR), without the interference produced by other users. We observed that this method presented better results than $p_{i,s} = 1$ or other combinations. After the initialization point has been established, steps 3 to 6th are repeated until the determined powers do not vary under an error value.

In step 3, the powers are normalized by the user power. The purpose of this is to introduce some weight in the constraints, that emphasizes the need of the beamformer to cancel more or less the interference in the subcarriers. After that, with these parameters fixed, in step 4 the problem can be solved with convex optimization tools. The new determined beamformers will carry after step 4 the power of the user, while the normalized $\rho_{i,s}$ will carry the distribution in the subcarriers.

In step 5 to 6, the purpose is to redistribute the power among the subcarriers, while holding the beamformers fixed. For this reason, the beamformers are first normalized by the user power (given by the trace), and then the power is redistributed among users and subcarriers. This new linear problem can be solved also with convex optimization tools.

Algorithm 4.1 Algorithm based on semidefinite optimization

1. Initialize $p_{i,s} = \gamma_{i,s} \frac{\sigma_{i,s}^2}{\mathrm{Tr}\{\mathbf{R}_{i,s}\}}$
2. **repeat**
3. Normalize the powers: $\rho_{i,s} = \frac{p_{i,s}}{\sum_{m=1}^{N} p_{i,m}} \cdot N$
4. Solve for each user with convex optimization tools

$$\begin{aligned}
&\min \sum_{i=1}^{K} \sum_{s=1}^{N} \mathrm{Tr}\{\mathbf{W}_i\} \\
&s.t.\, \rho_{i,s} \mathrm{Tr}\{\mathbf{R}_{i,s}\mathbf{W}_i\} - \gamma_{i,s} \sum_{n \neq i} \rho_{n,s} \mathrm{Tr}\{\mathbf{R}_{i,s}\mathbf{W}_n\} \geq \gamma_{i,s} \qquad i \in \{1, 2 \ldots K\} \\
&\quad \mathbf{W}_i = \mathbf{W}_i^H \qquad s \in \{1, 2 \ldots N\} \\
&\quad \mathbf{W}_i \succeq 0
\end{aligned}$$

5. Normalize the beamformer matrices: $\mathbf{U}_i = \frac{\mathbf{W}_i}{\mathrm{Tr}\{\mathbf{W}_i\}}$
6. Solve the linear optimization problem

$$\begin{aligned}
&\min \sum_{i=1}^{K} \sum_{s=1}^{N} p_{i,s} \\
&s.t.\, p_{i,s} \mathrm{Tr}\{\mathbf{R}_{i,s}\mathbf{U}_i\} - \gamma_{i,s} \sum_{n \neq i} p_{n,s} \mathrm{Tr}\{\mathbf{R}_{i,s}\mathbf{U}_n\} \geq \gamma_{i,s} \quad i \in \{1, 2 \ldots K\} \\
&\quad p_{i,s} \geq 0
\end{aligned}$$

7. **until** $\sum_{i=1}^{K} \sum_{s=1}^{N} \|p_{i,s}(t) - p_{i,s}(t+1)\|^2 \leq \epsilon$
8. Obtain the beamformer and power allocation as

$$\begin{aligned}
\mathbf{u}_i &= \max \text{ eig } \mathbf{W}_i \quad i \in \{1, 2 \ldots K\} \\
p_{i,s} &= \rho_{i,s} \cdot \mathrm{Tr}\{\mathbf{W}_i\} \quad s \in \{1, 2 \ldots N\}
\end{aligned}$$

Convergence of this algorithm is guaranteed, because at each step power is minimized. Again, the solution obtained will be a local minimum, since the final distribution of powers depends on the starting point. Also, a solution will only be reached when the first step reaches a feasible solution, depending again on this first point.

Last, we tried a different power allocation method with fixed beamformers, instead of the one proposed in step 6. In it, the intention was to distribute the power among the subcarriers of the user, and not between users.

$$\begin{aligned}
&\min t \\
&s.t.\, p_{i,s}\mathrm{Tr}\left\{\mathbf{R}_{i,s}\mathbf{U}_i\right\} - \gamma_{i,s}\sum_{n\neq i} p_{n,s}\mathrm{Tr}\left\{\mathbf{R}_{i,s}\mathbf{U}_n\right\} \geq \gamma_{i,s} \\
&\sum_{s=1}^{N} p_{i,s} = t \qquad\qquad i \in \{1,2\ldots K\} \\
&p_{i,s} \geq 0
\end{aligned} \tag{4.2}$$

In the end, we observed this power allocation provided worse results than the one used in step 6, of algorithm 4.1.

Cognitive radio

This algorithm uses a convex formulation of the original problem when fixing one of the variables and trying to optimize on the other. This fact allows to add new constraints to the problem that would still hold the problem convex. As explained in Section 2.3, if we add linear constraints that establish the maximum interference allowed to the primary users, we will be able to extend the previous algorithm to cover this new scenario with primary and secondary users.

In equation (4.3) we show the interference every primary user receives on each subcarrier. In the next step we make use of the semidefinite relaxation done in the algorithm and obtain a linear constraint as intended. Finally, this equation (4.4) is added to the set of constraints in steps 4 and 6 in algorithm 4.1, and the rest stays the same. In Chapter 6 we present some simulations that use this algorithm. Note we use the covariance matrix from the base station to the l th primary user on subcarrier s.

$$P_l = \sum_{i=1}^{K} p_{i,s}\mathbf{u}_i^H\mathbf{R}_{l,s}\mathbf{u}_i \leq \epsilon_{l,s} \tag{4.3}$$

$$\sum_{i=1}^{K} p_{i,s}\mathrm{Tr}\left\{\mathbf{R}_{l,s}\mathbf{U}_i\right\} \leq \epsilon_{l,s} \qquad l = 1,2\ldots L \tag{4.4}$$

$$s = 1,2\ldots N$$

4.2 Algorithm based on power control

In this section we present an algorithm that finds a suboptimal solution by trying to solve the equivalent uplink formulation. As in Section 2.2.1, we introduce a similar lemma, that extends the formulation to the OFDM case with individual constraints on all subcarriers.

Lemma 2. *If $\{\mathbf{w}_i\}$ are the optimal beamformers for the downlink beamforming problem 4.1 and $\{\mathbf{u}_i\}$ is the optimal solution for the following problem*

$$\min \sum_{i=1}^{K}\sum_{s=1}^{N} \rho_{i,s}$$

$$s.t. \frac{\mathbf{u}_i^H \rho_{i,s} \mathbf{R}_{i,s} \mathbf{u}_i}{\mathbf{u}_i^H \left(\sum_{n\neq i} \rho_{n,s}\gamma_{n,s}\mathbf{R}_{n,s} + \mathbf{I}\right)\mathbf{u}_i} \geq 1 \qquad i \in \{1,2\ldots K\} \tag{4.5}$$

$$\rho_{i,s} \geq 0 \qquad s \in \{1,2\ldots N\} \tag{4.6}$$

$$\|\mathbf{u}_i\|^2 = 1$$

then for all the beamformers, $\mathbf{w}_i = \sqrt{p_{i,s}}\mathbf{u}_i$ for some positive constants $p_{i,s}$.

Proof of this lemma can be found in Section 4.2.

The algorithm is similarly structured as the one in Section 2.2.1, where the beamformers are found by solving a generalized eigenvalue problem, and then the powers are updated until convergence. A summary of the algorithm can be checked in table 4.2. In table 4.4 a more detailed version is provided, with all the variables involved.

The notation used is as follows: $\mathrm{SINR}_{i,s}^{\mathbf{u}_i(t)} = \frac{\mathbf{u}_i^H \rho_{i,s}^t \mathbf{R}_{i,s}\mathbf{u}_i}{\mathbf{u}_i^H\left(\sum_{n\neq i}\rho_{n,s}^t\gamma_{n,s}\mathbf{R}_{n,s}+\mathbf{I}\right)\mathbf{u}_i}$ represents the equivalent uplink SINR using beamformer $\mathbf{u}_i(t)$ and correspondant power allocation. However some auxiliary variables have been used as middle steps, and we have represented it with the superindex aux, as $\mathrm{SINR}_{i,s}^{aux}$ and $\rho_{i,s}^{aux}(t)$. The index t represents the iteration step in the algorithm.

Algorithm 4.2 Algorithm based on power control

1. Initialization of beamformers and powers
2. **repeat**
3. Find the beamformers that satisfy
$$\mathrm{SINR}_{i,s}^{\mathbf{u}_i(t+1)} \geq \mathrm{SINR}_{i,s}^{\mathbf{u}_i(t)}$$
4. Update powers $\rho_{i,s}^{aux}(t+1) = \frac{1}{\mathrm{SINR}_{i,s}^{\mathbf{u}_i(t+1)}}\rho_{i,s}^{aux}(t)$, until $\mathrm{SINR}_{i,s} \approx 1$
5. **until** $\sum_{i=1}^{K}\sum_{s=1}^{N}\left\|\rho_{i,s}^{t+1} - \rho_{i,s}^{t}\right\|^2 \leq \epsilon$
6. After convergence, obtain the power vector

The initialization steps, in table 4.2 and 4.4 involve some difficulties that must be solved. The starting values of $\alpha_{i,s}$ cannot be randomly chosen, but have to guarantee that all SINRs of all subcarriers are of the same order. If for example, on some distribution of $\alpha_{i,s}$ the obtained beamformer does not cancel the interference on one subcarrier, the power allocation algorithm might diverge (being an infeasible beamformer), causing the whole algorithm to fail. Because of that, we must ensure the beamformer is feasible on all subcarriers, and we propose one initialization method that overcomes this problem. In table 4.3 we simply ensure that the beamformer equally balances the SINR on all subcarriers.

Algorithm 4.3 Initialization of the beamformers

1. Initialize $\rho_{i,s}^{0} = 1, \quad 1 \leq i \leq K$ and $1 \leq s \leq N$
2. Initialize $\alpha_{i,s}^{t} = \alpha_{i,s}^{t+1} = 1, \quad 1 \leq i \leq K$ and $1 \leq s \leq N$

Find the first beamformer:

1. **repeat**
2. Solve the GEV and get the $\mathbf{u}_i$ from the biggest eigenvalue, $\quad 1 \leq i \leq K$

$$\sum_{s=1}^{N} \alpha_{i,s}^{t} \rho_{i,s}^{0} \mathbf{R}_{i,s} \cdot \mathbf{u}_i = \sum_{s=1}^{N} \alpha_{i,s}^{t} \left(\sum_{n \neq i} \rho_{n,s}^{0} \gamma_{n,s} \boldsymbol{R}_{n,s} + \mathbf{I} \right) \mathbf{u}_i$$

3. Determine $\mathrm{SINR}_{i,s}^{aux} = \frac{\mathbf{u}_i^H \rho_{i,s}^{t} \mathbf{R}_{i,s} \mathbf{u}_i}{\mathbf{u}_i^H \left(\sum_{n \neq i} \rho_{n,s}^{t} \gamma_{n,s} \mathbf{R}_{n,s} + \mathbf{I} \right) \mathbf{u}_i}, \quad 1 \leq i \leq K$ and $1 \leq s \leq N$
4. **if** $\mathrm{SINR}_{i,s}^{aux} < \frac{1}{2} \max_{i,s} \left(\mathrm{SINR}_{i,s}^{aux} \right), \quad 1 \leq i \leq K$ and $1 \leq s \leq N$
5. Update $\alpha_{i,s}$: $\alpha_{i,s}^{t+1} \leftarrow \alpha_{i,s}^{t}$
6. **end if**
7. **until** $\mathrm{SINR}_{i,s}^{aux} \geq \frac{1}{2} \max_{i,s} \left(\mathrm{SINR}_{i,s}^{aux} \right)$ on all $\quad 1 \leq i \leq K$ and $1 \leq s \leq N$
8. $\mathrm{SINR}_{i,s}^{\mathbf{u}_i(1)} \leftarrow \mathrm{SINR}_{i,s}^{aux}$

Update the powers:

1. **repeat**

$$\rho_{i,s}^{aux}(t+1) = \frac{1}{\mathrm{SINR}(t)_{i,s}} \rho_{i,s}^{aux}(t) \quad \begin{array}{l} 1 \leq i \leq K \\ 1 \leq s \leq N \end{array}$$

2. **until** $\sum_{i=1}^{K} \sum_{s=1}^{N} \left\| \rho_{i,s}^{aux}(t+1) - \rho_{i,s}^{aux}(t) \right\|^2 \leq \epsilon$, i.e. $\mathrm{SINR}_{i,s} \approx 1$.
3. $\rho_{i,s}^{1} \leftarrow \rho_{i,s}^{aux}(t+1)$

After the algorithm is initialized, from table 4.2 steps 2 to 5 cover the main loop. In step 3, beamformers are searched that ensure the SINR they produce is better than the previous beamformer on all subcarriers. It is important to note that this fact will be key for convergence. The details on how we ensure this in the algorithm are shown in steps 6 to 11 in table 4.4. In them, we update the $\alpha_{i,s}$ to give more or less weight to the subcarriers in order to fulfill the property from before. The update formula of the $\alpha_{i,s}$ on step 4 of algorithm 4.3 and step 10 in 4.4 is also a key element in all the process. We propose two update formulas that worked well most of the times, but are not robust enough. For more robustness of the algorithm, a different update formula should be searched, as these ones will sometimes fail to converge.

$$\alpha_{i,s} \leftarrow \alpha_{i,s} + \left(\frac{\max_s \left(\mathrm{SINR}_{i,s}^{aux}\right) - \mathrm{SINR}_{i,s}^{aux}}{N_{subcarriers}} \right) \cdot \frac{1}{\mathrm{SINR}_{i,s}^{aux}} \tag{4.7}$$

$$\alpha_{i,s} \leftarrow \alpha_{i,s} \sqrt{\frac{\mathrm{SINR}_{i,s}^{aux}}{\mathrm{SINR}_{i,s}^{\mathbf{u}_i(t)}}} \tag{4.8}$$

The first equation tries to balance the difference of SINR's from the highest SINR and the subcarrier of interest, and proved to be a step small enough to converge on most of the scenarios (in nearly infeasible scenarios, the update could however fail). The second equation uses the last obtained SINR value, with the SINR of the previous step (which will be bigger if we are updating the α), and it also proved to converge. However this second approach seemed to be less robust. Note, that finding the right update is kind of an art, rather than technology, and therefore this algorithm might not be appropriate for a real implementation, as it presents some serious drawbacks.

The search of the beamformer must satisfy that the new beamformer does not penalize at each step any subcarrier. This is the reason behind updating the beamformer until $\mathrm{SINR}_{i,s}^{aux} < \mathrm{SINR}_{i,s}^{\mathbf{u}_i(t)}$ on all subcarriers. By doing so, we ensure that after this step the new beamformer will maximize the SINR of every subcarrier. The existence of a feasible beamformer is guaranteed if one feasible beamformer is found at the initialization process (for instance, that beamformer itself). However, there is no guarantee that the beamformer will converge towards an optimal solution.

The next step after a beamformer for each user is found, is to reallocate power on all subcarriers. In this process, step 4 in 4.2 and steps 13 to 14 in 4.4, a power minimization process is carried until all the individual constraints are satisfied with equality (being equal to one, as shown in the lemma). Convergence of this loop is guaranteed if a feasible point exists, as it is proven in [16].

Last, after convergence of the beamformers and $\rho_{i,s}$, the power vectors of the downlink problem must be determined. We followed a similar approach to equation (2.11), where we extended the notation to cover for the OFDM case. The power vector obtained will have positive elements, as it is shown in Subsection 4.2.

Algorithm 4.4 Details of the algorithm based on power control

Initialization: Follow steps from algorithm 4.3.

Main loop

1. **repeat**
2. $\rho_{i,s}^{t} \leftarrow \rho_{i,s}^{t+1}, \quad 1 \leq i \leq K$ and $1 \leq s \leq N$
3. $\mathrm{SINR}_{i,s}^{\mathbf{u}_i(t)} \leftarrow \mathrm{SINR}_{i,s}^{\mathbf{u}_i(t+1)}, \quad 1 \leq i \leq K$ and $1 \leq s \leq N$
4. Restore $\alpha_{i,s}$: $\alpha_{i,s}^{t} = \alpha_{i,s}^{t+1} = 1, \quad 1 \leq i \leq K$ and $1 \leq s \leq N$
5. **repeat**
6. $\alpha_{i,s}^{t} \leftarrow \alpha_{i,s}^{t+1}$
7. Solve the GEV and get the $\mathbf{u}_i$ from the biggest eigenvalue, $\quad 1 \leq i \leq K$
$$\sum_{s=1}^{N} \alpha_{i,s}^{t} \rho_{i,s}^{t} \mathbf{R}_{i,s} \cdot \mathbf{u}_i = \sum_{s=1}^{N} \alpha_{i,s}^{t} \left(\sum_{n \neq i} \rho_{n,s}^{t} \gamma_{n,s} \boldsymbol{R}_{n,s} + \mathbf{I} \right) \mathbf{u}_i$$
8. Determine $\mathrm{SINR}_{i,s}^{aux} = \frac{\mathbf{u}_i^H \rho_{i,s}^{t} \mathbf{R}_{i,s} \mathbf{u}_i}{\mathbf{u}_i^H \left(\sum_{n \neq i} \rho_{n,s}^{t} \gamma_{n,s} \mathbf{R}_{n,s} + \mathbf{I} \right) \mathbf{u}_i}, \quad 1 \leq i \leq K$ and $1 \leq s \leq N$
9. **if** $\mathrm{SINR}_{i,s}^{aux} < \mathrm{SINR}_{i,s}^{\mathbf{u}_i(t)}, \quad 1 \leq i \leq K$ and $1 \leq s \leq N$
10. Update $\alpha_{i,s}$: $\alpha_{i,s}^{t+1} \leftarrow \alpha_{i,s}^{t}$
11. **end if**
12. **until** $\sum_{i=1}^{K} \sum_{s=1}^{N} \left(\alpha_{i,s}^{t} - \alpha_{i,s}^{t+1} \right) = 0$, i.e. all $\mathrm{SINR}_{i,s}^{aux} \geq \mathrm{SINR}_{i,s}^{\mathbf{u}_i(t)}$
13. $\mathrm{SINR}_{i,s}^{\mathbf{u}_i(t+1)} \leftarrow \mathrm{SINR}_{i,s}^{aux}$
14. **repeat**
$$\rho_{i,s}^{aux}(t+1) = \frac{1}{\mathrm{SINR}_{i,s}^{\mathbf{u}_i(t+1)}} \rho_{i,s}^{aux}(t) \quad 1 \leq i \leq K, \quad 1 \leq s \leq N$$
15. **until** $\sum_{i=1}^{K} \sum_{s=1}^{N} \left\| \rho_{i,s}^{aux}(t+1) - \rho_{i,s}^{aux}(t) \right\|^2 \leq \epsilon$, i.e. $\mathrm{SINR}_{i,s} \approx 1$.
16. $\rho_{i,s}^{t+1} \leftarrow \rho_{i,s}^{aux}(t+1)$
17. **until** $\sum_{i=1}^{K} \sum_{s=1}^{N} \left\| \rho_{i,s}^{t+1} - \rho_{i,s}^{t} \right\|^2 \leq \epsilon$
18. After convergence: $\mathbf{p} = \mathbf{F}^{-1} \boldsymbol{\eta}$, where $\mathbf{p}$ and $\boldsymbol{\eta}$ are defined in equations (4.9) and (4.10), matrix $\mathbf{F}$ as
$$\mathbf{F} = \begin{bmatrix} \mathbf{D}_1 & \mathbf{G}_{1,2} & \dots & \mathbf{G}_{1,K} \\ \mathbf{G}_{2,1} & \mathbf{D}_2 & & \\ \vdots & & \ddots & \\ \mathbf{G}_{K,1} & \dots & & \mathbf{D}_K \end{bmatrix} \longrightarrow (N \cdot K \times N \cdot K)$$
and D_i and $G_{i,j}$ are matrices (4.13) and (4.15), respectively.

Proof of lemma 2

This lemma and its proof was taken from [3], where we introduced an extension in the notation. The lemma is defined in Section 4.2. To prove it we define the following vectors and matrices:

$$\mathbf{p} = \left[\underbrace{p_{1,1}\cdots p_{1,N}}_{N\,coeff.} \ \underbrace{p_{2,1}\cdots p_{2,N}}_{N\,coeff.}\cdots\underbrace{p_{i,1}\cdots p_{i,N}}_{N\,coeff.}\cdots p_{K,N} \right]^T \longrightarrow (N\cdot K\times 1) \quad (4.9)$$

$$\boldsymbol{\eta} = \left[\underbrace{\gamma_{1,1}\sigma_{1,1}^2\cdots\gamma_{1,N}\sigma_{1,N}^2}_{N\,coeff.} \underbrace{\gamma_{2,1}\sigma_{2,1}^2\cdots\gamma_{2,N}\sigma_{2,N}^2}_{N\,coeff.}\cdots\underbrace{\gamma_{i,1}\sigma_{i,1}^2\cdots\gamma_{i,N}\sigma_{i,N}^2}_{N\,coeff.}\cdots\gamma_{K,N}\sigma_{K,N}^2 \right]^T \quad (4.10)$$

$$\omega = \left[\underbrace{\|\mathbf{u}_1\|^2\ldots\|\mathbf{u}_1\|^2}_{N\,coeff.}\cdots\underbrace{\|\mathbf{u}_i\|^2\ldots\|\mathbf{u}_1\|^2}_{N\,coeff.}\cdots\|\mathbf{u}_K\|^2 \right]^T \longrightarrow (N\cdot K\times 1) \quad (4.11)$$

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_1 & 0 & \ldots & 0 \\ 0 & \mathbf{D}_2 & & \\ \vdots & & \ddots & 0 \\ 0 & \ldots & 0 & \mathbf{D}_K \end{bmatrix} \longrightarrow (N\cdot K\times N\cdot K) \quad (4.12)$$

$$\mathbf{D}_i = \begin{bmatrix} \mathbf{u}_i^H\mathbf{R}_{i,1}\mathbf{u}_i & 0 & \ldots & 0 \\ 0 & \mathbf{u}_i^H\mathbf{R}_{i,2}\mathbf{u}_i & & \\ \vdots & & \ddots & 0 \\ 0 & \ldots & 0 & \mathbf{u}_i^H\mathbf{R}_{i,N}\mathbf{u}_i \end{bmatrix} \longrightarrow (N\times N) \quad (4.13)$$

$$\mathbf{G} = \begin{bmatrix} 0 & \mathbf{G}_{1,2} & \cdots & \mathbf{G}_{1,N} \\ \mathbf{G}_{2,1} & 0 & & \vdots \\ \vdots & & \ddots & \mathbf{G}_{K-1,N} \\ \mathbf{G}_{K,1} & \cdots & \mathbf{G}_{K,N-1} & 0 \end{bmatrix} \longrightarrow (N\cdot K\times N\cdot K) \quad (4.14)$$

$$\mathbf{G}_{i,j} = \begin{bmatrix} \gamma_i\mathbf{u}_j^H\mathbf{R}_{i,1}\mathbf{u}_j & 0 & \ldots & 0 \\ 0 & \gamma_i\mathbf{u}_j^H\mathbf{R}_{i,2}\mathbf{u}_j & & \\ \vdots & & \ddots & 0 \\ 0 & \ldots & 0 & \gamma_i\mathbf{u}_j^H\mathbf{R}_{i,N}\mathbf{u}_j \end{bmatrix} \longrightarrow (N\times N) \quad (4.15)$$

where vector $\mathbf{p}$ represents the powers of all subcarriers stacked in a column of users, $\boldsymbol{\eta}$ presents the same structure, and $\boldsymbol{\omega}$ represents the norm of all beamformers (which will be unity norm). Submatrices $\mathbf{D}_i$ and $\mathbf{G}_{i,j}$ represent the power transmitted to user i on all subcarriers, and the interference that reaches it caused by user j, respectively. This notation allows us to represent problem

(4.1) in matrix form as

$$\begin{aligned}
&\min \mathbf{p}^T \boldsymbol{\omega} \\
&s.t.\ (\mathbf{D}-\mathbf{G})\,\mathbf{p} = \boldsymbol{\eta} \\
&\quad p_i \geq 0 \\
&\quad \|\mathbf{u}_i\| = 1
\end{aligned} \tag{4.16}$$

The lemma presented in Section 2.2.1 has a simpler matrix representation, but can be expressed in the same way as 4.16. From now on, we will follow the same steps as in [3] to show both lemmas. We will present a brief proof, but we suggest to check the reference for a deeper understanding on all steps.

Notice $\mathbf{D}$ is a diagonal matrix with independent elements, and therefore invertible, and that $\mathbf{D}^{-1}\mathbf{G}$ has only non-negative elements and is irreducible. Because of this we can use the Frobenius Perron theory for matrices with non-negative elements, and show that the spectral radius of $\mathbf{D}^{-1}\mathbf{G}$ is less than one (check [3]). This means

$$\lambda_{max}\left\{\mathbf{D}^{-1}\mathbf{G}\right\} < 1 \tag{4.17}$$

Using this last result, we then obtain that $\mathbf{I}-\mathbf{D}^{-1}\mathbf{G}$ is invertible and the inverse has only non-negative elements. Consequently

$$\mathbf{p} = \left(\mathbf{I}-\mathbf{D}^{-1}\mathbf{G}\right)^{-1}\mathbf{D}^{-1}\boldsymbol{\eta} \tag{4.18}$$

will be non-negative and a feasible point of (4.1). If we introduce the vector $\boldsymbol{\rho} = (\mathbf{D}-\mathbf{G})^{-T}\boldsymbol{\omega}$ we finally arrive at

$$\begin{aligned}
&\min \boldsymbol{\eta}^T \boldsymbol{\rho} \\
&s.t.\ \left(\mathbf{D}-\mathbf{G}^T\right)\boldsymbol{\rho} = \boldsymbol{\omega} \\
&\quad \rho_i \geq 0 \\
&\quad \|\mathbf{u}_i\| = 1
\end{aligned} \tag{4.19}$$

which can equivalently be written

$$\begin{aligned}
&\min \sum_{i=1}^{K}\sum_{s=1}^{N} \gamma_{i,s}\sigma_{i,s}^2\rho_{i,s} \\
&s.t.\ \mathbf{u}_i^H\left(\mathbf{I}-\rho_{i,s}\mathbf{R}_{i,s}+\sum_{n\neq i}\rho_{n,s}\gamma_{n,s}\mathbf{R}_{n,s}\right)\mathbf{u}_i \\
&\quad \rho_{i,s} \geq 0, \quad i = 1,2\ldots K, \quad s = 1,2\ldots N \\
&\quad \|\mathbf{u}_i\| = 1
\end{aligned} \tag{4.20}$$

This last result proves the one presented on the lemma, and generalizes that any beamformer from the equivalent uplink problem will be a feasible beamformer in the downlink.

4.3 Lagrange dual

In order to prove whether the algorithms we developed were optimal or not, we determined the Lagrangian of problem (4.1), and tried to solve the dual problem associated with it. For an explanation about the Lagrangian, its definition and properties, you may refer to Chapter 5 of [4]. Although we were not able to prove through the Lagrangian any kind of optimality, we believe its derivation can be at least instructive. We will explain in this section its derivation.

First we reformulate problem (4.1) into an equivalent one

$$\begin{aligned}
&\min \sum_{i=1}^{K} \sum_{s=1}^{N} \mathbf{w}_{i,s}^H \mathbf{w}_{i,s} \\
&s.t\, \frac{\mathbf{w}_{i,s}^H \mathbf{R}_{i,s} \mathbf{w}_i}{\sum_{n \neq i} \mathbf{w}_{n,s}^H \mathbf{R}_{i,s} \mathbf{w}_{n,s} + \sigma_{i,s}^2} \geq \gamma_{i,s}, \qquad \forall i \in \{1 \ldots K\} \\
&\left\| \frac{\mathbf{w}_{i,1}}{\|\mathbf{w}_{i,1}\|} - \frac{\mathbf{w}_{i,s}}{\|\mathbf{w}_{i,s}\|} \right\| = 0 \qquad \forall s \in \{1 \ldots N\}
\end{aligned} \tag{4.21}$$

Then we build the Lagrangian

$$\begin{aligned}
L &= \sum_i \sum_s \mathbf{w}_{i,s}^H \mathbf{w}_{i,s} - \sum_i \sum_s \lambda_{i,s} \left(\mathbf{w}_{i,s}^H \mathbf{R}_{i,s} \mathbf{w}_{i,s} - \gamma_{is} \sum_{n \neq i} \mathbf{w}_{n,s}^H \mathbf{R}_{i,s} \mathbf{w}_{n,s} - \gamma_{is} \sigma_{is}^2 \right) \\
&\quad + \sum_i \sum_{s \neq 1} \nu_{i,s} \left\| \frac{\mathbf{w}_{i,1}}{\|\mathbf{w}_{i,1}\|} - \frac{\mathbf{w}_{i,s}}{\|\mathbf{w}_{i,s}\|} \right\| \qquad (4.22) \\
&= \sum_i \sum_s \mathbf{w}_{i,s}^H \left(\mathbf{I} - \lambda_{i,s} \mathbf{R}_{i,s} + \sum_{n \neq i} \lambda_{n,s} \gamma_{n,s} \mathbf{R}_{n,s} \right) \mathbf{w}_{i,s} + \sum_i \sum_s \lambda_{i,s} \gamma_{i,s} \sigma_{i,s}^2 \\
&\quad + \sum_i \sum_{s \neq 1} \nu_{i,s} \left\| \frac{\mathbf{w}_{i,1}}{\|\mathbf{w}_{i,1}\|} - \frac{\mathbf{w}_{i,s}}{\|\mathbf{w}_{i,s}\|} \right\| \qquad (4.23)
\end{aligned}$$

and formulate the dual problem:

$$\max g(\lambda, \nu) = \max_{\lambda, \nu} \inf_{\mathbf{w}_{i,s}} L(\mathbf{w}_{i,s}, \lambda, \nu)$$

$$\max g(\lambda, \nu) = \begin{cases} +\infty & \left\| \frac{\mathbf{w}_{i,1}}{\|\mathbf{w}_{i,1}\|} - \frac{\mathbf{w}_{i,s}}{\|\mathbf{w}_{i,s}\|} \right\| \neq 0 \\ -\infty & \mathbf{I} - \lambda_{i,s} \mathbf{R}_{i,s} + \sum_{n \neq i} \lambda_{n,s} \gamma_{n,s} \mathbf{R}_{n,s} \not\succeq 0 \\ \max_\lambda \sum_i \sum_s \lambda_{i,s} \gamma_{i,s} \sigma_{i,s}^2 & s.t.\, \mathbf{I} - \lambda_{i,s} \mathbf{R}_{i,s} + \sum_{n \neq i} \lambda_{n,s} \gamma_{n,s} \mathbf{R}_{n,s} \succeq 0 \\ & \mathbf{w}_{i,1}^H \cdot \mathbf{w}_{i,1} \|\mathbf{w}_{i,s}\|^2 - \mathbf{w}_{i,1}^H \cdot \mathbf{w}_{i,s} \|\mathbf{w}_{i,1}\| \|\mathbf{w}_{i,s}\| = 0 \end{cases} \tag{4.24}$$

The first case shows the infeasibility of the problem if the vectors are not linearly dependent. The second case gives the trivial solution when the matrix in the quadratic term is not positive semidefinite. The third case is the only feasible and non trivial solution when the other two constraints are fulfilled. We can not show whether strong duality is hold or not. However, one would expect that the optimum of this problem may not have the same cost as the original problem with one beamformer per subcarrier. From this fact, strong duality would not hold.

Chapter 5

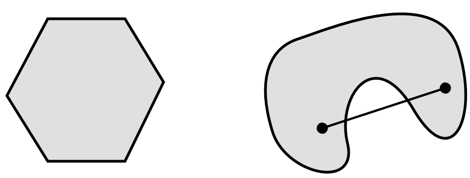
Transmit beamforming with general constraints

5.1 Introduction to quasiconvex analysis

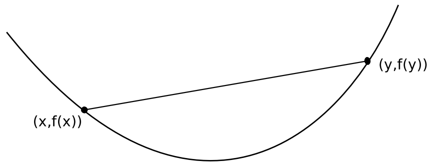
In this Subsection we would like to give a brief description about convex functions and quasiconvex analysis. The purpose of this is to give a small insight to the reader who is not used to working with these concepts, and help him understand the proposed algorithm. However, this short description will not be enough to explain the basics of convex analysis, and we present it only as an overview of what we are going to use in the following sections. Please, refer to the bibliography in [4] for a further understanding of the topic, where we based all of the information presented in here.

5.1.1 Definition of convexity and quasiconvexity

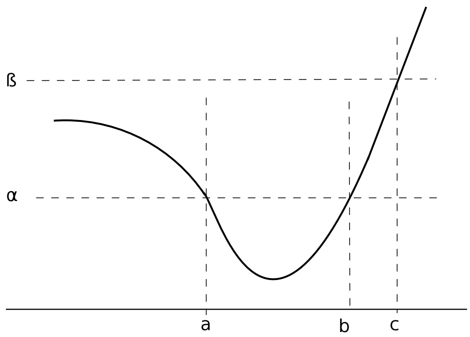
Convex set: A set C is convex if the line segment between any two points of the set lies in C. Roughly speaking a set is convex if every point in the set can be seen by every other point. On the following image, the hexagon on the left is convex. The kidney shaped set is not convex, since the line segment between the two points is not included in the set.



Convex function: A function f is convex if its domain is a convex set, and for all x, $y \in \mathbf{dom}\, f$, and $0 \leq \theta \leq 1$, we have $f\left(\theta x + (1-\theta)\, y\right) \leq \theta f\left(x\right) + (1-\theta)\, f\left(y\right)$. Geometrically it means that the line segment between $(x, f(x))$ and $(y, f(y))$ lies above the graph of f.



Quasiconvex function: A function f is quasiconvex if its domain and all its sublevel sets are convex. The sublevel sets are defined as the values of x that fulfill $S_\alpha = \{x \in \mathbf{dom}\, f | f(x) \leq \alpha\}$. A function is quasiconcave if $-f$ is quasiconvex, and quasilinear if f is quasiconvex and quasiconcave. All convex functions have convex sublevel sets, and therefore are quasiconvex, but the inverse is not true. In the following image we see an example of a quasiconvex function with drawn sublevels α and β which form the sublevel set S_α in the interval $[a, b]$ and the set S_β in the interval $(-\infty, c]$.



As a whole, convex problems are those whose objective function and inequality constraints are convex (equality constraints must be affine) and it can be proven that any local optimal point is also globally optimal. There exist very efficient methods to work with these problems, like interior-point methods, which can solve them within a specified accuracy and bounded number of iterations. However, this is not the case for quasiconvex problems, where some workaround must be used to be able to solve them. In these cases we may encounter local optima that are not the global solution, and our ability to solve them will rely on finding a set of convex functions that represent the sublevel sets of the quasiconvex ones. In the next subsection we will explain in more detail what needs to be done in those cases.

5.1.2 Quasiconvex optimization

Quasiconvex problems arise when the objective function (function to minimize) or some of the constraints are quasiconvex, and the rest are convex or affine. If the objective function is quasiconvex, then *bisection method* can be used; on the other hand, if the constraints are quasiconvex, then they can be replaced with equivalent convex constraints. Refer to the bibliography in [4] for a further description of this method and examples.

We will focus in the solution with quasiconvex constraints, which will be the problem we will try to solve in the subsequent sections. The way to solve an optimization problem with these constraints, is to find a convex function that represents the sublevel sets of the quasiconvex function (which are convex sets), and substitute them in the original problem. The family of functions we are looking for must satisfy for all $x \in \boldsymbol{R}^n$:

$$f(x) \leqslant t \iff \phi_t(x) \leq 0 \tag{5.1}$$

where $f(x)$ is quasiconvex and $\phi_t(x)$ is convex. If we are able to find such a function, we will substitute it in the original problem and we will obtain a convex problem that can be solved.

Now, we will introduce an interesting property we will later use in the formulation of our problem. That property is applied to the *perspective of a function*, defined with domain $\mathbf{dom}\, g = (x,t)|x/t \in \mathbf{dom}\, f,\ t > 0$ as

$$g(x,t) = t \cdot f\left(\frac{x}{t}\right) \tag{5.2}$$

where, this new function g is convex (concave) if f is convex (concave) and $t > 0$. This result is also extended to linear fractional arguments, where

$$g(\mathbf{x}) = \left(\mathbf{c}^T\mathbf{x} + \mathbf{d}\right) \cdot f\left(\frac{\mathbf{A}\mathbf{x} + \mathbf{b}}{\mathbf{c}^T\mathbf{x} + \mathbf{d}}\right) \tag{5.3}$$

will preserve the convexity of f providing $\mathbf{c}^T\mathbf{x} + \mathbf{d} > 0$. We will use this result in the next section.

5.2 Effective SINR constraints

Through our research to find a good way to represent the QoS of the secondary users, we considered the effective SINR mapping method to represent the performance of the different states an OFDM channel can have. We described the properties of this method in Section 3.2.3.1 (in the literature in [15]) and decided to use as *information measure* the exponential function. This method is called EESM (Exponential Effective SINR Mapping) which will be used in the following. In the next subsections we will introduce a suboptimal algorithm to solve the problem with EESM constraints, and later will discuss its optimality.

5.2.1 Problem formulation and algorithm proposed

The original formulation of the problem is

$$\begin{aligned} &\min \sum_{i=1}^{K} \sum_{s=1}^{N} \mathbf{w}_{i,s}^H \mathbf{w}_{i,s} \\ &s.t. \; -\beta \log \left(\frac{1}{N} \sum_{s=1}^{N} \exp \left(-\frac{1}{\beta} \frac{\mathbf{w}_{i,s}^H \mathbf{R}_{i,s} \mathbf{w}_{i,s}}{\sum_{n \neq i} \mathbf{w}_{n,s}^H \mathbf{R}_{i,s} \mathbf{w}_{n,s} + \sigma_{i,s}^2} \right) \right) \geq \gamma_i \quad \forall i \end{aligned} \tag{5.4}$$

where we made a minimization of power for all the users, and introduced some general constraints on each user based on the effective SINR mapping method. Overall, we have the logarithm of an average sum of exponentials, whose arguments are fractional functions with quadratic expressions on the numerator and denominator. As it stays, the problem is nonconvex and we will try to reformulate it as a convex one.

To simplify the problem we will consider the beamformers as stochastic matrices and relax the **rank** $= 1$ constraint, as we explained in Section 2.2.2. The relaxation we make does not produce an equivalent problem, but by running several simulations we observed that all obtained solutions on each iteration step had in fact **rank** $= 1$. This observation can be found in Section 2.2.2 and in the literature in [3, 2]. After this transformation, and eliminating the logarithm by taking its inverse, we obtain:

$$\begin{aligned} &\min \sum_{i=1}^{K} \sum_{s=1}^{N} \mathrm{Tr}\left[\mathbf{W}_{i,s}\right] \\ &s.t. \; \sum_{s=1}^{N} \exp \left(-\frac{1}{\beta} \frac{\mathrm{Tr}\left[\mathbf{R}_{i,s} \mathbf{W}_{i,s}\right]}{\sum_{n \neq i} \mathrm{Tr}\left[\mathbf{R}_{i,s} \mathbf{W}_{n,s}\right] + \sigma_{i,s}^2} \right) \leq N \cdot \exp\left(-\frac{\gamma_i}{\beta}\right) \\ &\quad \mathbf{W}_{i,s} \succeq 0 \qquad \forall i, s \\ &\quad \mathbf{W}_{i,s} = \mathbf{W}_{i,s}^H \end{aligned} \tag{5.5}$$

The problem is now quasiconvex, with convex objective function (in fact, linear) and quasiconvex constraints in the inequality set. The matrices $\mathbf{W}$ are hermitian and semipositive definite, and we relaxed the **rank** $= 1$ constraint. The quasiconvexity of the constraints is justified because the summation of positive weighted quasiconvex functions is quasiconvex, and the composition of a quasiconvex function (the exponential) with linear fractional functions as argument is also quasiconvex (the trace of a matrix is an affine expression).

As explained in Section 5.1.2 we will try to find some convex functions that represent the sublevel sets of the quasiconvex constraints. Using the definitions of the previous section we will name the functions $f_i(x)$ and $\phi_i(x)$ as the quasiconvex and convex functions respectively (the index i refers to the user i); however, we will change the general variable x to the beamforming matrix variable $\mathbf{W}$, only as a matter of notation.

In the next explanation, we will first find an equivalent convex function to the exponentials (which are quasiconvex due to the linear fractional argument). Then, we will average these functions to obtain the final equivalent constraint. We will call these functions with the subindex s, to differentiate them from the user final expression. For instance,

$$f_{i,s}(\mathbf{W}) = \exp\left(-\frac{1}{\beta}\frac{\mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{i,s}\right]}{\sum_{n\neq i}\mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{n,s}\right]+\sigma_{i,s}^2}\right) \tag{5.6}$$

and the original constraint

$$f_i(\mathbf{W}) = \sum_{s=1}^{N} f_{i,s}(\mathbf{W}) \leq N \cdot \exp\left(-\frac{\gamma_i}{\beta}\right) \tag{5.7}$$

Knowing that the perspective of a function preserves convexity (as explained in 5.1.2) and noting that the argument of each of the exponentials is a linear-fractional function, by multiplying each of the exponentials with the denominator of its argument we will obtain a convex function. One requirement is that the denominator is positive, and it can be proven that it will always be, since the noise variance is positive and the trace of two multiplying semidefinite matrices is nonnegative (page 52 in [4]).

The function we found however will not represent all the sublevel sets of $f_{i,s}(\mathbf{W})$, but will serve as approximation of it (this is discussed in Section 5.2.3). Then, the obtained convex functions will be

$$\phi_{i,s}(\mathbf{W}) = \left(\sum_{n\neq i}\mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{n,s}\right]+\sigma_{i,s}^2\right)\exp\left(-\frac{1}{\beta}\frac{\mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{i,s}\right]}{\sum_{n\neq i}\mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{n,s}\right]+\sigma_{i,s}^2}\right) \tag{5.8}$$

and the user convex constraint

$$\phi_i(\mathbf{W}) = \sum_{s=1}^{N} \phi_{i,s}(\mathbf{W}) \leq N \cdot \exp\left(-\frac{\gamma_i}{\beta}\right) \tag{5.9}$$

The next step is to scale each weighted exponential with some coefficient $\alpha_{i,s}$, so that it reduces the effect of the new introduced factor. Finally, the convex expression for user i is:

$$\sum_{s=1}^{N}\alpha_{i,s}\left(\sum_{n\neq i}\mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{n,s}\right]+\sigma_{i,s}^2\right)\exp\left(-\frac{1}{\beta}\frac{\mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{i,s}\right]}{\sum_{n\neq i}\mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{n,s}\right]+\sigma_{i,s}^2}\right) \leqslant N \cdot \exp\left(-\frac{\gamma_i}{\beta}\right) \tag{5.10}$$

Algorithm 5.1 Solution with EESM constraints

1. Initialize all $\alpha_{i,s} = 1$
2. **repeat**
3. Update $\alpha_{i,s} = \frac{1}{\sum_{n \neq i} \mathrm{Tr}[\mathbf{W}_{n,s}\mathbf{R}_{i,s}] + \sigma^2}$
4. Solve with convex optimization tools

$$\min \sum_{i=1}^{K} \sum_{s=1}^{N} \mathrm{Tr}\left[\mathbf{W}_{i,s}\right] \tag{5.11}$$

$s.t.$

$$\sum \alpha_{i,s} \left(\sum_{n \neq i} \mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{n,s}\right] + \sigma_{i,s}^2 \right) \exp\left(-\frac{1}{\beta} \frac{\mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{i,s}\right]}{\sum_{n \neq i} \mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{n,s}\right] + \sigma_{i,s}^2} \right) \leq N \cdot \exp\left(-\frac{\gamma_i}{\beta} \right)$$

$$\mathbf{W}_{i,s} \succeq 0 \quad \forall i, s$$

$$\mathbf{W}_{i,s} = \mathbf{W}_{i,s}^H$$

5. **until** $\alpha_{i,s} - \frac{1}{\sum_{n \neq i} \mathrm{Tr}[\mathbf{W}_{n,s}\mathbf{R}_{i,s}] + \sigma^2} < \epsilon$ for all $i \in \{1 \ldots K\}$ and $s \in \{1 \ldots N\}$

The algorithm we propose consists in solving the original problem with these convex constraints instead of the quasiconvex ones for some initial values of $\alpha_{i,s}^0$. Then, update the factors with $\alpha_{i,s}^{n+1} = \frac{1}{\left(\sum_{n \neq i} \mathrm{Tr}[\mathbf{R}_{i,s}\mathbf{W}_{n,s}] + \sigma_{i,s}^2\right)_n}$ at every step n, until conversion. The algorithm will end when $\alpha_{i,s}^{n+1} \cdot \left(\sum_{n \neq i} \mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{n,s}\right] + \sigma_{i,s}^2 \right)_{n+1} = 1$ so that the convex constraint equals at its optimum the original function.

Through our simulations we observed that the algorithm converged after three or four iterations, and as previously stated, the matrices $\mathbf{W}$ all had $\mathbf{rank} = 1$. To obtain the final beamformers from these matrices, the method explained in Section 2.2.2 or in the literature in [3] can be used.

We present the pseudocode in algorithm 5.1.

5.2.2 Cognitive radio

This algorithm can also be extended to work on the cognitive radio case as the ones from sections 2.3 and 4.1. As before, it uses a convex formulation and therefore, it is possible to add new constraints that hold the problem convex.

In equation (5.12) we show the interference every primary user receives on each subcarrier. In the next step we make use of the semidefinite relaxation done in the algorithm and obtain a linear constraint. Finally, this equation (5.13) is added to the set of constraints in step 4 in algorithm 5.1. In Chapter 6 we present some simulations. Note we use the channel covariance matrix of the primary user l on subcarrier s.

$$P_i = \sum_{i=1}^{K} p_{i,s} \mathbf{u}_i^H \mathbf{R}_{l,s} \mathbf{u}_i \leq \epsilon_{l,s} \tag{5.12}$$

$$\sum_{i=1}^{K} p_{i,s} \operatorname{Tr} \{\mathbf{R}_{l,s} \mathbf{U}_i\} \leq \epsilon_{l,s} \qquad l = 1, 2 \ldots L \tag{5.13}$$
$$s = 1, 2 \ldots N$$

5.2.3 Discussion of optimality

The condition of optimality would be proven if we showed

$$f_i (\mathbf{W}) \ \leqslant \ N \cdot \exp\left(-\tfrac{\gamma_i}{\beta}\right) \quad \Longleftrightarrow \quad \phi_i (\mathbf{W}) \ \leqslant \ N \cdot \exp\left(-\tfrac{\gamma_i}{\beta}\right) \tag{5.14}$$

for all $\mathbf{W} \in \boldsymbol{R}^{n \times n}$ and that $\phi_i (\mathbf{W}) \leq t \Rightarrow \phi_i (\mathbf{W}) \leq r$ for $r \geq t$, where $f_i (\mathbf{W})$ is quasiconvex and $\phi_i (\mathbf{W})$ is convex. However, this condition is not satisfied, and we will prove that. First, we should notice that the implication must be satisfied in all feasible points of the equation, and not only in the feasible points of the whole problem (the intersection of the feasible points of all constraints). Having that in mind, we may find a solution that satisfies $f_i (\mathbf{W})$ and does not satisfy $\phi_i (\mathbf{W})$.

Lets consider a set of beamformers that satisfies both $f_i (\mathbf{W})$ and $\phi_i (\mathbf{W})$ with equality (for example the optimal solution of the algorithm). We may increase the power transmitted to the user of interest in one of the exponentials ($\mathbf{W}_{i,s}$) so that the new set of equations is satisfied with strict inequality on both equations. Then, we might increase the interference created by other user (some $\mathbf{W}_{n,s}$) until $f_i (\mathbf{W}) = N \cdot \exp\left(-\frac{\gamma_i}{\beta}\right)$ is again satisfied with equality. However, while we increase the interference, in $\phi_i (\mathbf{W})$ the growth is both on the argument of the exponential and on the linear multiplying factor. This new set of beamformers will satisfy $f_i (\mathbf{W}) \leq N \cdot \exp\left(-\frac{\gamma_i}{\beta}\right)$ with equality, but will not satisfy $\phi_i (\mathbf{W}) \nleq N \cdot \exp\left(-\frac{\gamma_i}{\beta}\right)$.

So, what is the interpretation of our algorithm? What we are doing, is finding a convex function whose minimum is a feasible point of the original problem, but we do not know if that point is the optimal solution, or is any other point. However, we have observed that the convergence of the algorithm towards the final solution is a fix-point that does not depend on the initialization of the $\alpha_{i,s}$. Furthermore, we observed analytically, that the algorithm with N=1 subcarriers yielded the optimal solution from Section 2.2. On the contrary, we cannot extend that result to the general case. Therefore, whether this fix-point is the optimal solution or not is left for future work.

5.2.4 Implementation in CVX

As we mentioned before in Section 2.2.2, we used CVX [10], to solve many of the convex problems we encountered. However, this package does not give support to represent the perspective of a function in the specifications of the problem, due to the *Disciplined Convex Programming* ruleset [10, 9]. Because of that, we had to implement the perspective of the functions to use in CVX ourselves. We present the solution we adopted, because the reader might need it if he wants to implement the algorithm.

The function we want to implement in CVX is $f\left(x\right) = t \cdot f\left(\frac{x}{t}\right)$, provided $t > 0$. To be able to represent it we form the epigraph of the function as:

$$\text{epi} f\left(x\right) = \left\{ \left(x, t, z\right) \mid t > 0,\, t \exp\left(\frac{x}{t}\right) \leq z \right\} \tag{5.15}$$

$$\begin{aligned} t \exp\left(\frac{x}{t}\right) &\leq z \\ \ln t + \frac{x}{t} &\leq \ln z \\ t \ln t + x - t \ln z &\leq 0 \\ t \ln \frac{t}{z} + x &\leq 0 \end{aligned}$$

In CVX environment, when we declare an epigraph variable, the program will minimize that variable without specifying it directly. In the CVX User Guide, they advice to use it only when declaring functions, and not in normal problems. What we did in the last set of equations, was to declare the epigraph variable, and convert the problem to one that CVX can solve. The last expression contains the relative entropy function, already present in the set of functions provided.

The final code is shown in table 5.1.

Table 5.1: Perspective of the exponential

```
function cvx_optval = perspective_exp( x,t )
%PERSPECTIVE_EXP Perspective function of the Exponential.
% PERSPECTIVE_EXP(x,t), where both x and t are affine expressions,
% computes t*exp(x/t), if t is strictly positive, and +Inf
otherwise.
cvx_begin
  epigraph variable z
  rel_entr(t,z)+x <= 0;
cvx_end
```

5.3 Capacity constraints

In this section we present the approach we followed to solve problem (5.16), and why this solution did not converge. Again, this problem reads

$$\begin{aligned}
&\min \sum_{i=1}^{K}\sum_{s=1}^{N} \mathbf{w}_{i,s}^H \mathbf{w}_{i,s} && (5.16)\\
&s.t.\ \frac{1}{2}\sum_{s=1}^{N} \log\left(1+\frac{\mathbf{w}_{i,s}^H \mathbf{R}_{i,s}\mathbf{w}_{i,s}}{\sum_{n\neq i}\mathbf{w}_{n,s}^H \mathbf{R}_{i,s}\mathbf{w}_{n,s}+\sigma_{i,s}^2}\right) \geq \gamma_i \\
&\qquad\qquad\qquad\qquad\qquad\qquad i \in \{1,2\ldots K\}
\end{aligned}$$

The same as problem (5.4) with effective SINR constraints, the problem is nonconvex. Again we can use the semidefinite relaxed approach to transform the problem to quasiconvex,

$$\begin{aligned}
&\min \sum_{i=1}^{K}\sum_{s=1}^{N} \mathrm{Tr}\left[\mathbf{W}_{i,s}\right] && (5.17)\\
&s.t.\ \frac{1}{2}\sum_{s=1}^{N} \log\left(1+\frac{\mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{i,s}\right]}{\sum_{n\neq i}\mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{n,s}\right]+\sigma_{i,s}^2}\right) \geq \gamma_i \\
&\qquad \mathbf{W}_{i,s} \succeq 0 \qquad\qquad \forall i, s\\
&\qquad \mathbf{W}_{i,s} = \mathbf{W}_{i,s}^H
\end{aligned}$$

where we did not establish the **rank** = 1 constraint. Again, we can use the perspective of a function property to transform the quasiconvex problem into a convex one

$$\begin{aligned}
&\min \sum_{i=1}^{K}\sum_{s=1}^{N} \mathrm{Tr}\left[\mathbf{W}_{i,s}\right] && (5.18)\\
&s.t.\ \frac{1}{2}\sum_{s=1}^{N} \alpha_{i,s}\left(\sum_{n\neq i}\mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{n,s}\right]+\sigma_{i,s}^2\right)\log\left(1+\frac{\mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{i,s}\right]}{\sum_{n\neq i}\mathrm{Tr}\left[\mathbf{R}_{i,s}\mathbf{W}_{n,s}\right]+\sigma_{i,s}^2}\right) \geq \gamma_i \\
&\qquad \mathbf{W}_{i,s} \succeq 0 \qquad\qquad \forall i, s\\
&\qquad \mathbf{W}_{i,s} = \mathbf{W}_{i,s}^H
\end{aligned}$$

where we introduced again the coefficients $\alpha_{i,s}^{n+1} = \frac{1}{\left(\sum_{n\neq i}\mathrm{Tr}[\mathbf{R}_{i,s}\mathbf{W}_{n,s}]+\sigma_{i,s}^2\right)_n}$. However this approach did not converge, as the $\alpha_{i,s}$ would keep growing and avoiding conversion into a fix point. This is due to the interference term directly contributing to satisfy the constraints. The effect we observed on this equation, is that interference is encouraged to satisfy the constraints, making all users transmit in the same subcarrier to increase it. Of course, this behavior is unwanted and deviates from the expected solution of the original equation.

This other approach presented the same problems, and also failed to converge

$$
\begin{aligned}
&\min \sum_{i=1}^{K} \sum_{s=1}^{N} \operatorname{Tr}\left[\mathbf{W}_{i,s}\right] \\
&s.t.\ \frac{1}{2} \sum_{s=1}^{N} \left(\frac{1}{\sigma_{i,s}^2} \sum_{n \neq i} \operatorname{Tr}\left[\mathbf{R}_{i,s} \mathbf{W}_{n,s}\right] + 1 \right) \log \left(1 + \frac{\operatorname{Tr}\left[\mathbf{R}_{i,s} \mathbf{W}_{i,s}\right]}{\sum_{n \neq i} \operatorname{Tr}\left[\mathbf{R}_{i,s} \mathbf{W}_{n,s}\right] + \sigma_{i,s}^2} \right) - \alpha_{i,s} \geq \gamma_i \\
&\qquad \mathbf{W}_{i,s} \succeq 0 \qquad\qquad \forall i, s \\
&\qquad \mathbf{W}_{i,s} = \mathbf{W}_{i,s}^H
\end{aligned} \tag{5.19}
$$

where $\alpha_{i,s}^{n+1} = \left(\frac{1}{\sigma_{i,s}^2} \sum_{n \neq i} \operatorname{Tr}\left[\mathbf{R}_{i,s} \mathbf{W}_{n,s}\right] \right)_n$.

Again, to be able to solve these convex problems in CVX we had to define the perspective function of the logarithm. In this case the final code is shown in table 5.2

Table 5.2: Perspective of the logarithm

```
function cvx_optval = perspective_log( x,t )
%PERSPECTIVE_LOG Perspective function of the Logarithm.
% PERSPECTIVE_LOG(x,t), where both x and t are affine expressions,
% computes t*log(x/t), if t is strictly positive, and +Inf
otherwise.
cvx_begin
  hypograph variable z
  rel_entr(t,x)+z <= 0;
cvx_end
```

Chapter 6

Simulations and results

In this chapter we show some results and simulations using the algorithms previously described. We present two sections, one with the normal transmit beamforming, and the second considering the cognitive case.

6.1 Transmit beamforming

In this scenario we considered 3 users, 2 subcarriers and 6 antennas at the base station. The channel coefficients were randomly generated using ZMCS and i.i.d. Gaussian random variables. The covariance matrices were determined as $\mathbf{R}_{i,s} = \mathbf{h}_{i,s}\mathbf{h}_{i,s}^H$. The noise power at the receiver was set to 0.05 W, and the stopping criterion had been set to $\epsilon = 0.001$. All graphs were averaged over 200 simulations.

In figure 6.1, we represent the total power used function of the SINR. We computed the algorithms from sections 4.1 and 4.2 which required individual constraints on all subcarriers. The $\gamma_{i,s}$ constraints were set equal on all subcarriers and users, and ranged from -15 dB to 15 dB. Note that these algorithms used a unique beamformer at each user. In this figure, we also present the optimal algorithm from section 2.2.2 that uses one beamformer per subcarrier. This helps to set a lower bound on the achievable solution. In fact, this lower bound is the solution to the Lagrange dual problem explained in section 4.3.

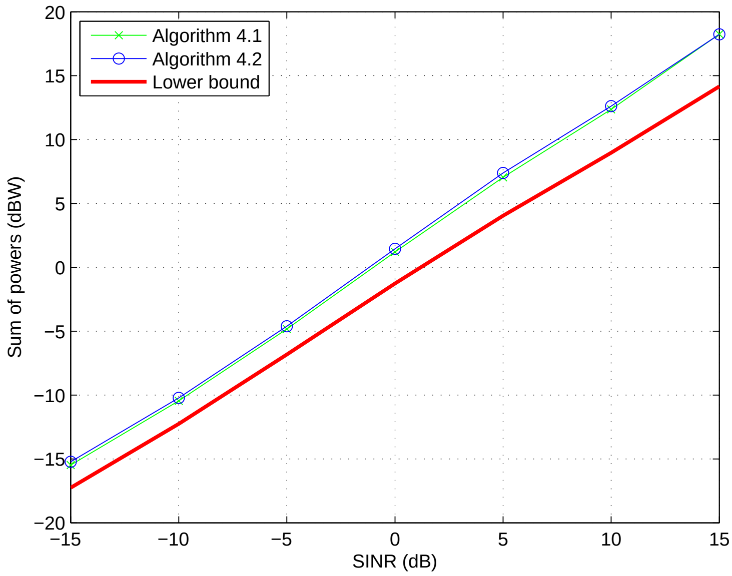


Figure 6.1: Algorithms with individual constraints

In figure 6.2, the algorithm shown is the one with effective SINR constraints from section 5.2. We represent again the sum of powers of all users, function of the effective SINR values. In this case, the user constraints γ_i were set equal on every user, and ranged from -15 dB to 15 dB to deploy the figure. We used $\beta = 1.69$ which is the recommended parameter for a QPSK modulation scheme and $2/3$ code rate [15].

We compare the algorithm with the optimal solution when using equal individual constraints on all subcarriers (algorithm from 2.2.2). Note, that with equal invididual constraints, they present the same *effective SINR* as the one used in the optimization problem. From this graph we observe a minor improvement when using algorithm 5.1, than with the other one. This is because this algorithm tries to make all the subcarrier SINR similar, due to all having same noise power. However this behaviour may change when having different noise variances.

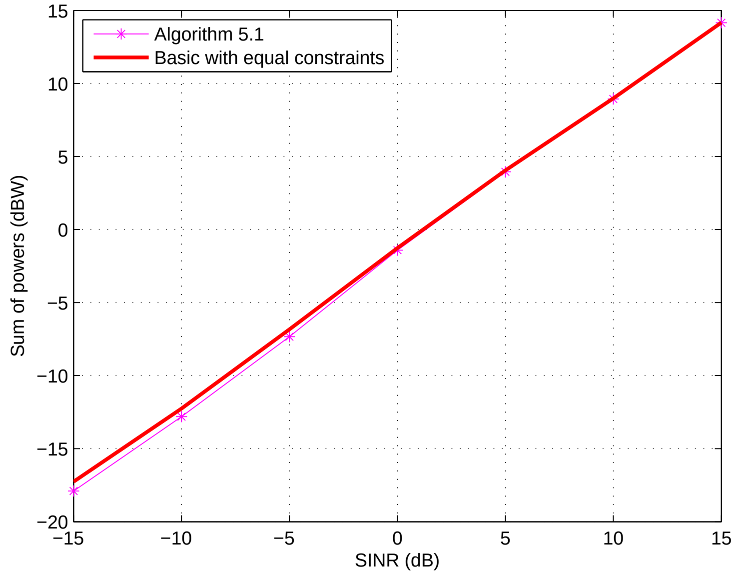


Figure 6.2: Algorithm with effective SINR constraints

Finally, in figure 6.3 the conversion of powers function of the number of iterations is presented. It can be observed that after the third iteration all of them have converged. These curves correspond to the previously explained algorithms from sections 4.2, 4.1 and 5.2, respectively.

It is worth noting that the algorithm based on power control adjusts the total power after the third iteration, while the algorithm based on semidefinite optimization barely adjusts some decimals. Therefore, with this algorithm the precision acquired is normally enough after the first iteration. On the other side, the curve with the effective SINR algorithm starts at an unfeasible point in the first iteration, and then quickly converges to its solution after the third iteration. This behavior depends on the initialization point of the algorithm (selection of $\alpha_{i,s}$ in algorithm table 5.1) .

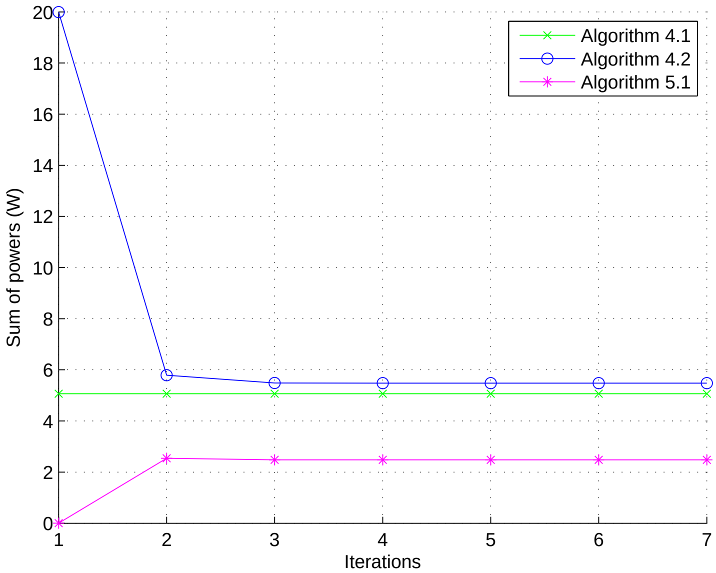


Figure 6.3: Number of iterations

6.2 Cognitive radio

In this case, a similar scenario was used with 3 secondary users, 2 subcarriers and 6 antennas at the base station. On the other hand, the presence of one primary user was added. The total amount of interference allowed to this user was of 0.1 W. The same parameters as 0.05 W of noise powers, $\epsilon = 0.001$ as stopping criterion and 200 simulations to make the average was used. The $\gamma_{i,s}$ will also have the range form -15 dB to 15 dB.

On figure 6.4 the algorithm based on semidefinite optimization with individual constraints and fixed beamformers is presented, and is compared to the optimal algorithm which uses different beamformers on each subcarrier as lower bound (section 2.3) .

In figure 6.5, the algorithm with EESM constraints is presented, and is again compared to the algorithm with equal individual constraints.

Finally, in figure 6.6 we show the conversion of powers, function of the iteration values.

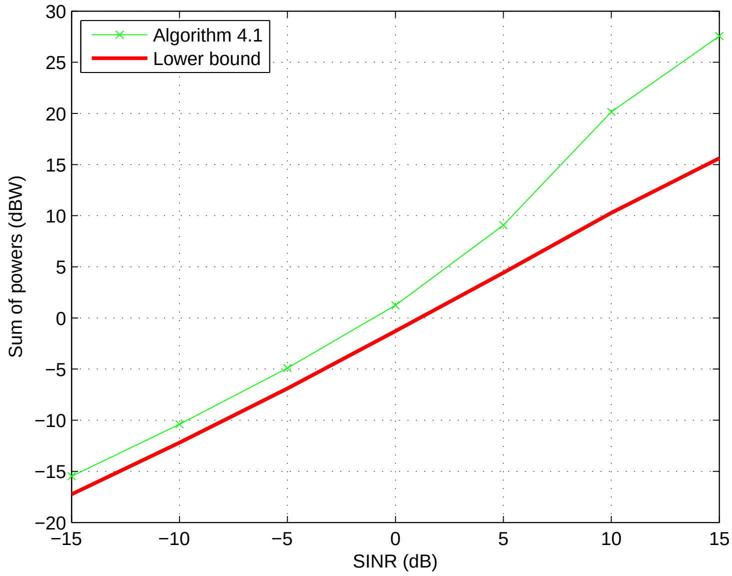


Figure 6.4: Algorithms with individual constraints in the CR case

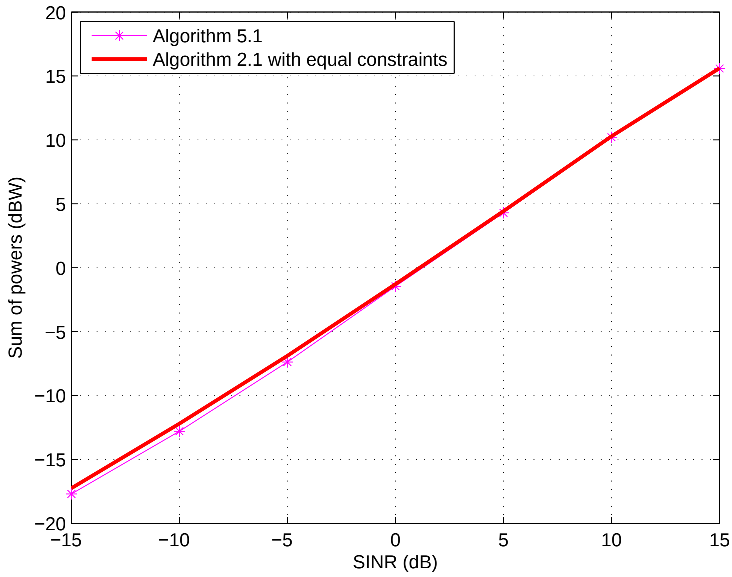


Figure 6.5: Algorithm with effective SINR constraints in the CR case

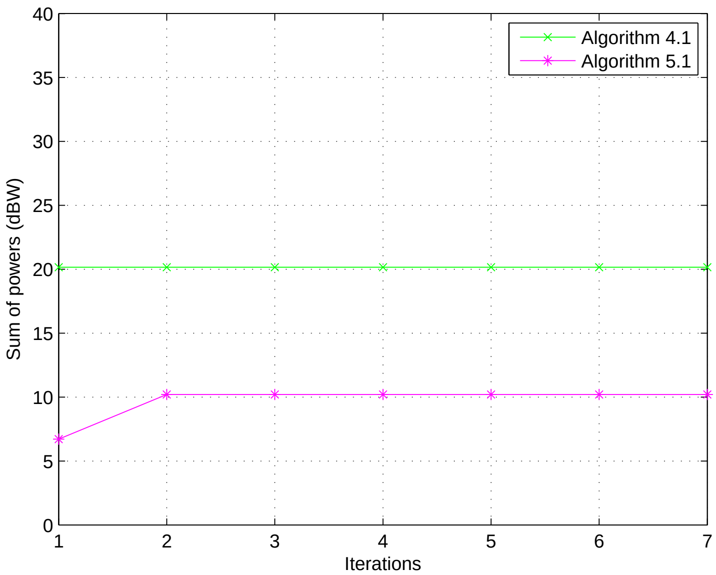


Figure 6.6: Number of iterations in the CR case

Conclusion

Transmit beamforming in downlink OFDM scenarios present a difficult optimization problem on resource allocation. Not only should the beamformers be found, but also the power distribution in the spectrum to satisfy some user QoS requirement be determined. Previous works explained in Chapter 2 and in [3] describe optimal algorithms that allocate these resources on the single channel case. In this thesis we proposed other solutions that considered resource allocation in the different user subcarriers.

In Chapter 2 we presented the scenario description of the problem at hand, and briefly explained the previous results in the single channel case. Then, in Chapter 3 we explained the motivation to extend these results to the OFDM case, and introduced two problem formulations to consider. Both of these formulations minimize power at the base station, where one requires individual constraints on every subcarrier and the other only one constraint per user.

In Chapter 4 we proposed two algorithms that suboptimally solved the problem with specific constraints on every subcarrier. For reasons of signaling reduction between the base station and the receivers, we required the users to use only one beamformer on all subcarriers, while satisfying some SINR constraints. The simulation of these algorithms were shown in Chapter 6, where we represented the total power function of the individual SINR's. In addition to all this, we showed that one of these algorithms could be extended to cover the cognitive radio approach.

Finally, in Chapter 5 we dealt with the problem formulation with general constraints. We proposed one algorithm that finds a feasible solution with Exponential Effective SINR Mapping (EESM) constraints, and discussed its optimality. We observed the algorithm always converged to a fix point regardless of the initialization point, and left further conclusions about its optimality as future work.

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